A Locomotive Strategy for a Stair-Climbing Mobile Platform Based on a New Contact Angle Estimation

H. Hong, D. Kim, J. Kim, J. Oh and H. S. Kim

Abstract— This paper presents a locomotive strategy for a stair-climbing mobile platform built on a rocker-bogie platform, which can surmount indoor stairs or steps by virtue of a new contact angle estimation method. First, link parameters of the rocker-bogie mechanism are optimized through the Taguchi method, with the aim of making the trajectory of the center of mass (CM) of the mobile platform as smooth as possible. Based on the optimization result, the mobile platform is compactly designed to suit indoor applications. A simple, robust and cost-effective estimation method is then proposed to obtain information of the contact angle between each wheel and the stairs with high fidelity. A composite locomotion strategy is suggested, based on the kinematic and kinetic analysis, combined with the contact angle estimation. The stair-climbing capability of the proposed mobile platform is successfully validated through extensive experiments.

I. INTRODUCTION

The mobile platform plays an important role in various applications, such as simple transport equipment, autonomous mobile machines and electric wheelchairs. The capability of overcoming obstacles such as steps or stairways guarantees excellent accessibility to indoor environments frequently encountered in many applications, so that this capability stands out as the most important factor among others. There have been manifold locomotive mechanisms to overcome obstacles, and among those mechanisms, wheel-link mechanisms such as rocker-bogie [1, 2], ORF-L [3], CEDRA [4] and SHRIMP [5] are widely adopted, due to their simple configuration.

In the previous research, a stair-climbing mobile robot is optimally designed on the basis of the rocker-bogie mechanism. In order to improve both the mobile stability and the adaptability during climbing a stair, optimization via the Taguchi method was carried out [6]. Based on the previous optimization result, the rocker-bogie based mobile platform is compactly constructed, so that all of the motors and tilt angle sensors are placed inside its hollow links and main body, which helps to prevent undesired interference with indoor environments.

If all wheel velocities are the same during climbing up or down a stairway, wheel slips are likely to occur, so that rotational errors are inevitable. To eliminate these detrimental effects, each wheel velocity should be given differently, according to the mobile platform’s posture while climbing up or down a stairway. Kinematic analysis is performed to understand the relation between the wheel velocity and posture of the mobile platform. In addition, a simple, robust and cost-effective wheel contact estimation method between each wheel of the mobile platform and its circumstances is proposed. Then, a composite locomotive strategy is derived, by combining the optimal torque control based on the kinetic analysis, with the velocity control based on the kinematic analysis. The effectiveness of the proposed locomotive method is verified through various experiments, using different sizes of stairs.

This paper is organized as follows. In Section II, the details of the rocker-bogie based mobile platform are described, with the optimization result. Section III presents the simple, robust and cost-effective wheel contact angle estimation method, together with the kinematic analysis. The stair-climbing ability of the rocker-bogie mobile platform is verified with experiments in Section IV.

II. A ROCKER-BOGIE BASED MOBILE PLATFORM

A. Optimization with the Taguchi method

In order not only to improve stair-climbing ability, but also to prevent instability caused by sticking, slipping and floating of a wheel of the mobile platform, an optimization process via the Taguchi method was carried out in previous research. The goal of the optimization is to minimize the area between the CM trajectory and the straight line determined by the slope of the stair, which implies that the mobile platform enables efficient climbing up or down the stair, without falling down [6].

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According to the Taguchi method, the radius of 3 wheels \( (R_1, R_2, R_3) \) and the lengths of 4 links \( (l_1, l_2, l_3, l_4) \) are optimally chosen. A 2-D schematic diagram of the optimized rocker-bogie mechanism is shown in Fig. 1, whose detailed dimensions are given in Table I.

<table>
<thead>
<tr>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( l_4 )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>140 mm</td>
<td>70 mm</td>
<td>230 mm</td>
<td>194 mm</td>
<td>25 mm</td>
<td>60 mm</td>
<td>70 mm</td>
</tr>
</tbody>
</table>

### B. Mechanical design

Based on the optimization result described in the previous section, the mechanical design of the rocker-bogie mobile platform is performed with the aims of lightness, compactness, and minimization of undesired interference between the mobile platform and obstacles.

A side view of the CAD model is shown in Fig. 2(a), where the CM of the mobile platform corresponds to a revolute joint connected to the main body. As confirmed in Fig. 2(a), Link 1 and Link 2 are in the shape of a triangle, with a streamlined edge to avoid interference while climbing a stairway, and all motors, tilt angle sensors and wires are installed inside the hollow Links.

![Figure 2](image_url)

**Figure 2.** (a) Side view of CAD model for the rocker-bogie platform, and (b) detailed configuration of the proposed rocker-bogie platform

Fig. 2(b) describes the details of the proposed mobile platform, whose total weight and overall size are 14.5kg and 560mm × 460mm × 305mm (width × height × length), respectively. Wall detecting sensor units are attached near the front wheels, to precisely detect a contact situation during the climbing of stairs. Electrical cylinders are connected between the main body and Link 2, in order to maintain the main body horizontally during the climbing of stairs. The detailed specifications of motors, actuators and sensors used for the rocker-bogie mobile platform are summarized in Table II.

### III. Control Strategy

#### A. Kinematic analysis

During climbing up or down stairs, each wheel velocity must be controlled separately, to prevent rotational errors or slips. Due to the symmetry of the mobile platform as well as the simplicity of analysis, the 2-D kinematic model in Fig. 3(a) is adopted [7, 8], where \( \{F\} \) is a fixed reference frame, and a moving frame \( \{B\} \) is allocated to the passive revolute joint corresponding to the CM of the mobile platform. \( \{H\} \) and \( \{S\} \) are moving frames allocated to the joint \( A \), but \( \{H\} \) is parallel with \( \{B\} \) and Link 2, while \( \{S\} \) rotates with Link 1. \( \{A_i\} \) is allocated at the end of the link, and parallel to the bottom line of each link. \( \{W_i\} \) is positioned at the center of each wheel, and rotates with each wheel. \( \{C_i\} \) is allocated to the contact point between wheels and ground. The geometric relations between moving frames \( \{W_i\}, \{A_i\} \) and \( \{C_i\} \) are illustrated in detail in Fig. 3(b).

![Figure 3](image_url)

**Figure 3.** (a) Coordinate frame systems for the proposed rocker-bogie mechanism, and (b) detailed coordinate frame systems for the \( i^{th} \) wheel
Based on the previous coordinate frame systems, the following transformation matrices are obtained.

\[
\begin{bmatrix}
1 & 0 & 0 & -\Delta \sin \theta_{20} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -\Delta \cos \theta_{20} \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad
\begin{bmatrix}
\cos \theta_{20} & 0 & \sin \theta_{20} & 0 \\
0 & 1 & 0 & 0 \\
-\sin \theta_{20} & 0 & \cos \theta_{20} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

where,

\[\begin{aligned}
\theta_2 &= \frac{\pi}{2} (\theta_1 - \theta_2 + \beta), \quad \beta = \tan^{-1}\left(\sqrt{\frac{\beta^2 + \gamma^2}{\alpha^2 + \gamma^2}}\right)
\end{aligned}\]

Eq. (1) shows the kinematic relations among frames \{B\}, \{H\} and \{S\}. \Delta and \theta_{20} represent the distance between the origins of the frames \{B\} and \{H\}, and the rotational angle between the frames \{H\} and \{S\}, respectively. Since the frames \{S\}, \{A_1\} and \{A_2\} are parallel to each other, and the frames \{H\} and \{A_i\} are parallel to each other, the transformation matrices are derived as follow:

\[
\begin{bmatrix}
1 & 0 & 0 & \frac{z_d_{A_1}}{2} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & \frac{z_d_{A_2}}{2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

Finally, the transformation matrices among the frames \{A_i\}, \{W_i\} and \{C_i\}, \text{i}=1, 2, 3 can be expressed as below:

\[
\begin{bmatrix}
\cos (\frac{\pi}{2} - \theta - \beta) & 0 & -\sin (\frac{\pi}{2} - \theta - \beta) & 0 \\
0 & 1 & 0 & 0 \\
\sin (\frac{\pi}{2} - \theta - \beta) & 0 & \cos (\frac{\pi}{2} - \theta - \beta) & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(3)

Let us define the wheel velocity \(\dot{q}_i\), \text{i}=1, 2, 3 and the robot body velocity \(\pi \dot{p}_b\) as

\[
\pi \dot{p}_b = \left[\pi \dot{V}_{b_1}, \pi \dot{V}_{b_2}, \pi \dot{V}_{b_3}, \pi \omega_b\right]^T, \quad \dot{q}_i = \lambda \omega_{b_i}
\]

(4)

According to the approach in [9], the Jacobian matrix \(J_1\) for the wheel \(W_1\) can be derived as follows:

\[
\begin{bmatrix}
\cos \theta_1 & \sin \theta_1 & 0 & \frac{d_{A_1}}{2} \\
0 & 1 & 0 & 0 \\
-\sin \theta_1 & \cos \theta_1 & 0 & \frac{d_{A_2}}{2} \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(5)

Since \(c_{V_{C_1}} = R\sin \gamma_1 \cdot \lambda \omega_{b_1}\) and \(c_{V_{C_2}} = R\sin \gamma_1 \cdot \lambda \omega_{b_1}\), Eq.(5) can be recast as

\[
\pi \dot{p}_b = \begin{bmatrix}
R_1(\cos \gamma_1 \cos \theta_1 + \sin \gamma_1 \sin \theta_1) - \frac{d_{A_1}}{2} \\
R_1(\sin \gamma_1 \cos \theta_1 + \cos \gamma_1 \sin \theta_1) - \frac{d_{A_2}}{2} \\
R_2(\cos \gamma_2 \cos \theta_2 + \sin \gamma_2 \sin \theta_2) - \frac{d_{A_3}}{2} \\
1
\end{bmatrix}
\]

(6)

In a similar manner, the Jacobian matrix \(J_2\) and \(J_3\) for the wheel \(W_2\) and \(W_3\) can be derived below:

\[
\pi \dot{p}_b = \begin{bmatrix}
R_2(\cos \gamma_2 \cos \theta_2 + \sin \gamma_2 \sin \theta_2) - \frac{d_{A_2}}{2} \\
R_2(\sin \gamma_2 \cos \theta_2 + \cos \gamma_2 \sin \theta_2) - \frac{d_{A_3}}{2} \\
R_3(\cos \gamma_3 \cos \theta_3 + \sin \gamma_3 \sin \theta_3) - \frac{d_{A_3}}{2} \\
1
\end{bmatrix}
\]

(7)

Combining Eqs. (6)-(8), the kinematic equation of the proposed mobile platform can be derived as follows:

\[
A_0 \pi \dot{p}_b = J_0 \dot{q}
\]

(9)

Therefore, the wheel velocity \(\dot{q}_i\) required to generate the desired robot velocity \(\pi \dot{p}_b\) may be given by

\[
\dot{q} = (I - J_0)^{-1} J_0 A_0 \pi \dot{p}_b
\]

(10)

B. Contact angle estimation

With the Jacobian matrix \(J_0\) in (9), each wheel velocity can be controlled, according to the posture of the mobile platform while climbing stairs; but note that the contact angles between wheels and ground are required to compute the Jacobian matrix. Since it is not easy to find out the wheel contact angles directly, various contact angle estimation methods have been proposed. Axle-mounted force sensors, a camera-based vision system equipped with an extended Kalman filter [10, 11], and kinematic estimators to use the kinematic relation between wheel velocities and the pitch angle of the mobile platform [12] are examples; but they still suffer from inevitable sensor noise and unexpected wheel slip, so that their performance seems unsatisfactory for real applications.

In order to effectively cope with such shortcomings of previous methods, a simple, robust and cost-effective method is proposed in this section, from the key observation that the two tilt angles \(\theta_1\) and \(\theta_2\) of the mobile platform are closely related to each other during the climbing of stairs. Fig. 4(a) shows the relation between \(\theta_1\) and \(\theta_2\) while climbing a stair of 300mm × 100mm (tread × riser). When climbing a step, the
tilt angle trajectory consists of \(a \rightarrow b \rightarrow c \rightarrow P \rightarrow a\). In contrast, when climbing more than two steps, the tilt angle trajectory consists of \(a \rightarrow b \rightarrow c \rightarrow P \rightarrow b \rightarrow c\), where \(c \rightarrow P \rightarrow b \rightarrow c\) is repeated while continuing to climb stairs. Recall that the previous sequence is reversed when the mobile platform climbs down.

As shown in Fig. 4(b), the tilt angle trajectory may change, according to the size of stair. To make a look-up table, a reference point is possible to construct a look-up table that can relate the tilt angles with the contact angle in a one-to-one manner for each size of stair. To make a look-up table, a reference point is selected as the current reference point from the look-up table. The previous estimation procedure will be carried out for both left- and right-side links of the mobile platform, and at every iteration step, and the difference between the identified current reference points of left- and right-side links must be checked and maintained within the bound, by appropriately adjusting each wheel velocity or torque. Although the applications of this method may be limited to stair/step climbing wheel-type mobile robots, this method is quite simple and inexpensive, compared to other contact angle estimation methods. Since the sizes of steps or stairs are limited by Building Codes, the computational burden as well as the storage requirement may be considerably reduced.

C. Control scheme for stair-climbing

In the previous research [6], a friction requirement metric \(\mu_{eq}\) was defined, to evaluate the stair-climbing ability of the mobile platform. The kinetic analysis in [6] was carried out to find out the minimum moment of each wheel required to escape from situations where the mobile platform cannot climb the stair, and halts because of a slip. As a result, the minimally required torque \(\tau_i^*, i=1, 2, 3\) for the \(i^{th}\) wheel can be calculated, by using the friction requirement metric \(\mu_{eq}\)

\[
\tau_i^* = \mu_{eq} R_i N_i
\]

where, \(R_i\) and \(N_i\) correspond to the radius and the normal force of the \(i^{th}\) wheel, respectively (for more details, refer to section 4 of [6]).

In contrast to the case of climbing up stairs, where the main focus is to prevent slip, accurate position control of the mobile platform is the most important issue when climbing down stairs. When the mobile platform climbs down stairs, reaction forces, except for normal forces, are relatively small enough to be negligible, so that minimum torque is required, to prevent falling down the stair. In order to accurately control the mobile robot’s motion, a velocity control method based on the kinematic analysis in Section III is adopted while climbing down the stair.

![Figure 4. Tilt angles trajectories during climbing a step or a stair of (a) 300mm × 100 mm, and of (b) 300mm × 100mm, 310mm × 160 mm, and 300mm × 200 mm](image)

![Figure 5. Diagram of the control scheme for the mobile platform](image)
which are fed to the left and right drives equipped with a PID-based position control system [13]. Through the CAN communication, the computed velocity command $\dot{q}_i$ or the torque command $\tau_i^*$ is transferred to each motor drive, to control the motor velocity or the motor torque.

IV. VERIFICATION EXPERIMENT

In real applications, various sizes of stairs or steps may be encountered. Therefore, extensive experiments are carried out to validate the adaptability, as well as the mobile stability, of the proposed mobile platform, under indoor circumstances. Three types of stairs are prepared, whose tread and riser dimensions are $300\text{mm} \times 100\text{mm}$ (18.4°), $310\text{mm} \times 160\text{mm}$ (27.3°), and $300\text{mm} \times 200\text{mm}$ (33.8°), respectively.

Before performing stair-climbing experiments, driving tests, including forward/backward moving and circular path tracking, are also carried out, where the maximum speed of the proposed mobile platform is measured to be 22 m/min, and the obstacle overcoming capability is successfully examined with a bar of 80mm height.

A. Stair climbing capability test

Snapshots of climbing up three kinds of stair are shown in Fig. 6, where the width of each stair is slightly larger than that of the mobile platform, to check the straightness of motion while climbing the stair, which may be disturbed by wheel slip, rotation errors, sticking or floating, etc. As seen in Fig. 6, the proposed mobile platform enables the climbing up of all three stairs, without incorrect motion caused by slip or rotation errors. The maximum velocity of the proposed mobile platform is measured to be about 6 sec/step during climbing up the stair of $310\text{mm} \times 160\text{mm}$. Considering that the normal human walking speed is approximately 2 sec/step, the climbing speed of the mobile platform seems to be quite acceptable.

A driving test to verify the capability of climbing down the stairs is also carried out. Compared to the climbing up cases, there is no reaction force along the moving direction while climbing down, so that minimum torque is required to climb down stairs. Without using the velocity control in Section III, the mobile platform would fail in climbing down the stairs, and fall down. As a result, accurate position control becomes a more crucial issue in the case of climbing down the stair, which may be an important reason to select a velocity control scheme for the proposed mobile platform.

B. Comparison of CM trajectories

Additional experiments using a Motion Capture System are carried out, to obtain the real data for the CM trajectories, as well as the contact angles. Figs. 7(a) and 7(b) compare the real and simulated CM trajectories, while climbing up stairs of $300\text{mm} \times 100\text{mm}$ and $310\text{mm} \times 160\text{mm}$, respectively, where the black dotted line denotes the straight slope of the stair, and the red line and the blue dotted lines correspond to the real and the simulated CM trajectories, respectively. As shown in Fig. 7, the maximum errors between the real and the simulated CM trajectories are 8mm and 10mm for the stairs of $300\text{mm} \times 100\text{mm}$ and $310\text{mm} \times 160\text{mm}$, respectively. Considering the overall size of the stair, the accuracy of the simulated CM trajectory through kinematic relation, as well as mobile stability, are ensured through the experiments, because the real CM trajectory of the mobile robot does not undergo any drastic or discontinuous change.
C. Contact angle estimation

In this section, a comparison between the actual and estimated contact angles is given, to evaluate the performance of the proposed estimation method. The actual contact angle is acquired from calculation of the trajectories of wheels and body obtained from previous experiments using the motion capture system.

Figs. 8 and 9 show a comparison between the estimated contact angles and the actual ones, while climbing a step and more than two steps of a stair of 310mm × 160mm, where the blue dashed line and the red line represent the estimated contact angle and the real one, respectively. When the mobile platform climbs a step, the corresponding contact angles change, as depicted in Fig. 8; but when climbing more than two steps, the first and the third wheels move on the riser or the edge simultaneously, which explains the distinction of the third wheel’s contact angle trajectory shown in Fig. 9 (c). As confirmed in Figs. 8 and 9, the proposed contact angle estimation method proves its effectiveness, so that although the maximal estimation errors are approximated to be around 7° for both, the trajectory of the estimated contact angle is very similar to that of the real one.

V. CONCLUSION

The rocker-bogie based mobile platform is constructed by using optimized design parameters, with the aim of keeping the CM trajectory as smooth as possible. By placing sensors, motors and wires inside the hollow links, the mobile platform is compact, and so suitable for indoor applications; and also, undesired interference with stairs or external impact is effectively prevented. A simple, robust and cost-effective contact angle estimation method is then proposed, from the key observation that the relation between the tilt angles of the links is uniquely determined by the size of the stair. The high accuracy, as well as the robustness of the proposed estimation method, is shown through the extensive experiments. It is successfully proved that the proposed mobile platform can climb up and down various sizes of stairs autonomously, with the help of velocity and optimal torque control schemes.

REFERENCES