

# A new planar 3-DOF parallel mechanism with continuous 360-degree rotational capability<sup>†</sup>

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## Abstract

This paper presents a novel three degrees of freedom (3-DOF) planar parallel mechanism with a 360-degree rotational capability, which can be used as a positioning device. A high rotational capability is necessary to reduce process time and to apply the mechanism to various fields such as micro-machining and printed circuit board (PCB) depaneling. The proposed mechanism has the advantage of a continuous rotational capability over existing planar parallel mechanisms with limited rotation. The position and velocity kinematics are derived, and the singularities are analyzed based on the Jacobian matrices. The workspace can be determined not to have a singularity inside, and the kinematic variables for enlarging the singularity-free workspace are determined. The application example on the PCB depaneling process is proposed for future work.

**Keywords:** Kinematics; Planar parallel mechanism; Rotational capability; Three degrees of freedom

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## 1. Introduction

A parallel mechanism is one with a closed loop of several serial chains. Parallel mechanisms have more chains and less moving inertia than serial mechanisms, which leads to them having comparative advantages of higher speeds, better stiffness, and improved positioning accuracies [1]. Due to these advantages, parallel mechanisms have received increasing attention from industries and universities in recent decades.

However, from an application point of view, the parallel mechanism has a major drawback in its limited rotational capability. The Stewart-Gough mechanism, which is a well-known standard of the 6-DOF parallel mechanism, can only realize 30 degrees of rotation [2]. To overcome this drawback, Kim et al. suggested the Eclipse-I and -II mechanisms to be applied in a 5-axis machine tool and motion simulator

by realizing a 90-degree rotation and a 360-degree rotation, respectively [3, 4].

However, the Eclipse mechanisms are designed to be used in spatial motions, and thus they are inefficient when used exclusively in planar motions. The planar parallel mechanism has some comparative advantages over the stack-type serial mechanism with the *X-Y* stage motions, and the additional rotational axis results in error accumulation, moving inertia, and stiffness. Thus, several planar parallel mechanisms have also been proposed [5-11], but the rotational capability of these parallel mechanisms is limited. Therefore, the mechanisms are not suitable for some industrial applications such as micro-machining and PCB depaneling [12, 13], where a high rotational capability is necessary to reduce process time and to apply the mechanism to various tasks.

This paper presents a new planar 3-DOF mechanism capable of a continuous 360-degree rotation as well as *X-Y* translational motions. Figs. 1 and 2 illustrate the architecture and rotational capability of the mechanism, respectively. The proposed mechanism is

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motivated by the Eclipse mechanism, which has some advantages over the Eclipse-I mechanism. They are as follows.

- The mechanism can be designed not to have singularity configurations in the workspace, so it does not need over-actuation.
- The number of actuators can be reduced from eight to three to realize the planar motion.
- The stiffness can be enhanced by removing the spherical joint.
- The mechanism is more suitable for the design of a combined machine with serial mechanisms.

The proposed planar mechanism consists of 3-PPR (where P and R denote prismatic and revolute joints, respectively) serial sub-chains. As the first P denotes the actuated P moving along the fixed circular guide, continuous rotational motion can be realized. The three arrows in Fig. 1 denote the actuated circular Ps. Kinematics, Jacobian, singularity, and workspace

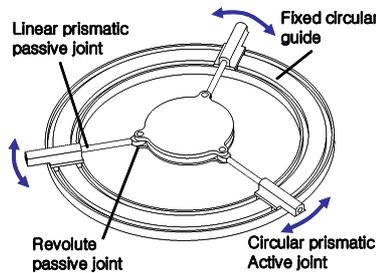


Fig. 1. Architecture of the novel 3-DOF planar mechanism.

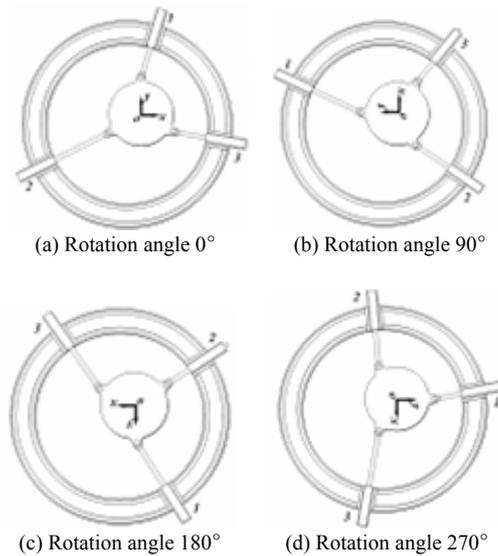


Fig. 2. Rotational capability of the mechanism at an arbitrary (x, y) position.

analyses are discussed based on the mechanism. A practical application example of PCB depaneling system is presented next.

## 2. Kinematics of the mechanism

This section presents the architecture of the mechanism, followed by the procedures describing its inverse and forward kinematics. The kinematics is solved based on the Cartesian coordinate, and polar coordinates can be used to avoid coordinate coupling.

### 2.1 Description of the mechanism

The proposed mechanism consists of 3-PPR serial sub-chains that move independently on a fixed circular guide. Fig. 3 shows the geometric configuration of the mechanism. The mechanism has 3-DOF and three actuated joints ( $A_i$ ) located along the circular guide. The connecting legs ( $A_iB_i$ ) are extensible. Each extensible leg consists of two links connected by a P. The inner end of the extensible legs is mounted to the moving platform via the three Rs ( $B_i$ ). The actuated Ps along the circular guide yield a continuous 360-degree rotational capability of the mechanism's platform at about the Z-axis in the fixed frame  $\{F\}$ , as shown in Fig. 3. The kinematic parameters of the mechanism are the fixed circular guide radius ( $r_a$ ) and the radius of the moving platform ( $r_b$ ). The actuated P values are referred to as  $\theta_i$  and the P values in the connecting legs as  $\rho_i$ , where  $i$  stands for the index of each serial sub-chain.

### 2.2 Inverse kinematics

The problem of inverse kinematics is concerned with determining the values of the actuated joints from the position and orientation of the moving frame  $\{M\}$ , which is attached to the moving platform. The inverse kinematics of the mechanism can be solved as a whole by solving the inverse kinematics of each sub-chain. The Cartesian positions of the Rs, which are denoted as  $B_i$  in Fig. 3, are calculated from the position ( $\bar{p}$ ) and orientation ( $\mathbf{R}$ ) of the moving platform in the fixed frame  $\{F\}$ .

$$\bar{b}_i = \mathbf{R}^M \bar{b}_i + \bar{p} \tag{1}$$

where  ${}^M\bar{b}_i$  is the vector of the R  $i$  expressed in the coordinates of the moving frame.  $\mathbf{R}$  and  $\bar{p}$  are expressed in the coordinates of the fixed frame and are defined as follows:

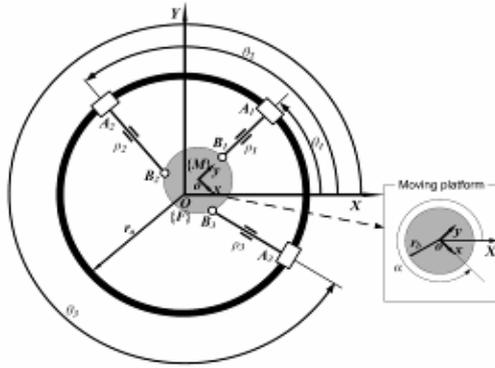


Fig. 3. Description of the mechanism and kinematic parameters.

$$\vec{p} = [x \ y]^T \tag{2}$$

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{3}$$

where  $\alpha$  is the angle of the moving frame with respect to the fixed frame. The actuated circular P values ( $A_i$ ) are then calculated from the positions of the Rs.

$$\theta_i = \arctan(b_{iy}, b_{ix}) \tag{4}$$

This can be expressed as

$$\theta_i = \arctan\left(\frac{{}^M b_{ix} \sin\alpha + {}^M b_{iy} \cos\alpha + y}{{}^M b_{ix} \cos\alpha - {}^M b_{iy} \sin\alpha + x}\right) \tag{5}$$

where  $\theta_i$  is the value of the circular P  $i$ , and  $b_{ix}$  and  $b_{iy}$  are the  $x$  and  $y$  coordinates of  $\vec{b}_i$ , respectively. The Cartesian positions of the circular Ps  $\vec{a}_i$  can be determined by their joint values, and the fixed circular guide radius as follows:

$$\vec{a}_i = [r_a \cos\theta_i \ r_a \sin\theta_i]^T. \tag{6}$$

Eqs. (5) and (6) yield eight solutions to the inverse kinematics of this mechanism because each circular P value ( $\theta_i$ ) has two values according to the choice of the  $\arctan(b_{iy}, b_{ix})$ . The solution can be uniquely determined from the initial configuration of the mechanism and the continuity of the solution.

Additionally, by using Eqs. (1) and (6), the passive P values ( $\rho_i$ ) in the connecting legs are determined as follows:

$$\rho_i = \|\vec{a}_i - \vec{b}_i\| \tag{7}$$

### 2.3 Forward kinematics

The problem of forward kinematics is determining the position and orientation of the moving frame given the actuated joint values. Generally, the forward kinematics solution for most parallel mechanisms is not unique and is difficult to determine analytically. For this mechanism, the forward kinematics solution is obtained by solving the forward kinematics of each sub-chain after calculating the actuated and passive joint values. The passive joint values must be determined numerically by the actuated joint values and the kinematics constraint equation.

The kinematics constraint equation, (Eq. 8), is defined for conditions in which the distances between the Rs of the moving platform are constant and equal to,  $\sqrt{3}r_b$ .

$$\vec{g}(\vec{\theta}_a, \vec{\theta}_p) = \begin{bmatrix} \|\vec{b}_1 - \vec{b}_2\|^2 - 3r_b^2 \\ \|\vec{b}_2 - \vec{b}_3\|^2 - 3r_b^2 \\ \|\vec{b}_3 - \vec{b}_1\|^2 - 3r_b^2 \end{bmatrix} = \vec{0} \tag{8}$$

where

$$\vec{b}_i = [(r_a - \rho_i) \cos\theta_i \ (r_a - \rho_i) \sin\theta_i]^T \tag{9}$$

$$\vec{\theta}_a = [\theta_1 \ \theta_2 \ \theta_3]^T \tag{10}$$

$$\vec{\theta}_p = [\rho_1 \ \rho_2 \ \rho_3]^T \tag{11}$$

The  $\vec{\theta}_p$  can be determined by a numerical method such as the Newton-Raphson method. The position of the moving platform ( $\vec{p}$ ), which is the center of an equilateral triangle, and the rotation matrix can then be calculated by using  $\vec{b}_i$  as follows:

$$\vec{p} = \frac{1}{3}(\vec{b}_1 + \vec{b}_2 + \vec{b}_3), \tag{12}$$

$$R = [\vec{R}_x \ \vec{R}_y], \tag{13}$$

where

$$\vec{R}_x = \frac{1}{\sqrt{3}r_b}(\vec{b}_3 - \vec{b}_2) \text{ and } \vec{R}_y = \frac{1}{r_b}(\vec{b}_1 - \vec{p}).$$

### 3. Jacobian matrix

This section presents a derivation of the Jacobian matrix, which is the relationship between the velocity of the active joints, the passive joints, and the end-

effector.

$$\dot{\bar{x}} = -\left(\frac{\partial \bar{q}}{\partial \bar{x}}\right)^{-1} \left( \left(\frac{\partial \bar{q}}{\partial \theta_a}\right) + \frac{\partial \bar{q}}{\partial \theta_p} \Phi \right) \dot{\theta}_a, \tag{19}$$

**3.1 Kinematic constraint Jacobian matrix**

The kinematic constraint Jacobian matrix, which is a relationship between the velocity of the active joints and that of the passive joints, can be determined by differentiating Eq. (8). It can be expressed as follows:

$$\frac{\partial \bar{g}}{\partial \theta} \dot{\theta} = \frac{\partial \bar{g}}{\partial \theta_a} \dot{\theta}_a + \frac{\partial \bar{g}}{\partial \theta_p} \dot{\theta}_p = \bar{0} \tag{14}$$

The velocity of the passive joints can then be determined as follows:

$$\dot{\theta}_p = -\left(\frac{\partial \bar{g}}{\partial \theta_p}\right)^{-1} \frac{\partial \bar{g}}{\partial \theta_a} \dot{\theta}_a, \tag{15}$$

where the kinematic constraint Jacobian is expressed as follows:

$$\Phi = -\left(\frac{\partial \bar{g}}{\partial \theta_p}\right)^{-1} \frac{\partial \bar{g}}{\partial \theta_a}. \tag{16}$$

**3.2 Forward Jacobian matrix**

The forward Jacobian matrix, which is the relationship between the velocity of the active joints and that of the end-effector, can be determined by differentiating Eq. (7), which can be expressed in vector form as follows:

$$\bar{q}_i(\bar{x}, \bar{\theta}_a, \bar{\theta}_p) = \begin{bmatrix} \|\bar{a}_1 - \bar{b}_1\|^2 - \rho_1^2 \\ \|\bar{a}_2 - \bar{b}_2\|^2 - \rho_2^2 \\ \|\bar{a}_3 - \bar{b}_3\|^2 - \rho_3^2 \end{bmatrix} = \bar{0} \tag{17}$$

where

$$\bar{x} = [x \ y \ \alpha]^T.$$

The resulting differentiated Eq. can then be expressed as follows:

$$\frac{\partial \bar{q}}{\partial \bar{x}} \dot{\bar{x}} + \frac{\partial \bar{q}}{\partial \theta_a} \dot{\theta}_a + \frac{\partial \bar{q}}{\partial \theta_p} \dot{\theta}_p = \bar{0}. \tag{18}$$

The velocity of the end-effector can then be determined by solving

where the forward Jacobian is expressed as

$$J_f = -\left(\frac{\partial \bar{q}}{\partial \bar{x}}\right)^{-1} \left( \left(\frac{\partial \bar{q}}{\partial \theta_a}\right) + \frac{\partial \bar{q}}{\partial \theta_p} \Phi \right). \tag{20}$$

**4. Singularity analysis**

A singularity is a configuration in which the number of DOF in the mechanism increases or decreases instantaneously. Singularities are one of the most significant and critical problems in the design and control of parallel mechanisms because they lead to loss of controllability, breakage, and degradation of stiffness in the mechanism.

Singularities can be classified into three types: configuration space, actuator, and end-effector [14]. A configuration space singularity is the intersection of all possible actuator singularities. Actuator singularities are configurations in which some links in the mechanism can move when the actuators are locked, or in which actuator forces cause the mechanism to break down. End-effector singularities are configurations in which the end-effector of the mechanism instantaneously loses mobility in one or more DOF. The three kinds of singularities occur in the following conditions, respectively:

$$\text{rank} \left( \frac{\partial \bar{g}}{\partial \theta}(\theta_j) \right) < 3 \tag{21}$$

$$\text{rank} \left( \frac{\partial \bar{g}}{\partial \theta_p}(\theta_j) \right) < 3 \tag{22}$$

$$\text{rank} (J_f(\theta_j)) < 3 \tag{23}$$

where

$$\bar{\theta} = [\bar{\theta}_a \ \bar{\theta}_p]^T.$$

To determine the singular configuration, the condition number, which is defined as the ratio of the maximum singular value to the minimum singular value, is computed in the workspace. As the matrices in Eqs. (21)-(23) are nonlinear simultaneous equations, the numerical method using the condition number is more suitable than calculating the analytical solution of the equations. In the analysis, the singular-

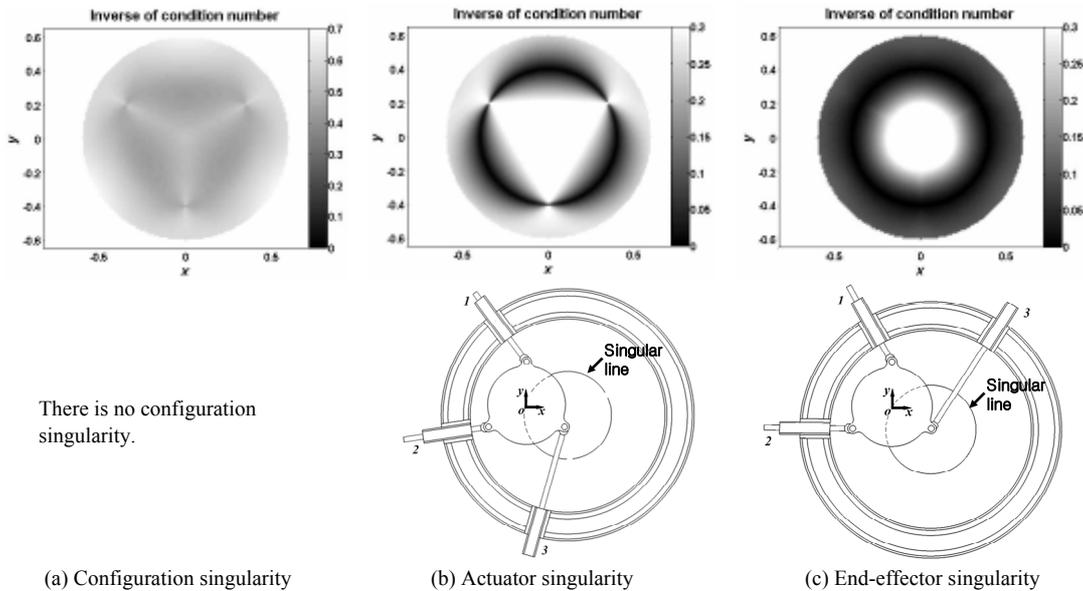


Fig. 4. Singularity analysis using the inverse of the condition number and an example of a singular configuration.

ity occurs when the inverse of the condition number equals zero. For calculation,  $r_b$  is assumed equal to  $0.4r_a$ . For all rotation angles of the moving platform, the same singular configurations exist in this mechanism. Accordingly, the rotation angle of the moving platform is specified as 0 degree.

The upper part of Fig. 4 shows the inverse of the condition number in the reachable workspace, and the bottom part shows the example of each singular configuration. In the entire workspace, there is no configuration space singularity, and the other two types of singularities (actuator and end-effector) coexist in the workspace. The actuator and end-effector singularities occur when the moving platform is located on the boundary of the circle with a radius of  $r_b$  and at the center of the fixed frame, which is shown in Figs. 4 (b) and (c). This excludes cases in which one of the moving platform's Rs is also located at the center.

**5. Workspace analysis**

The workspace of a mechanism refers to the full set of positions and orientations achievable by the moving frame. It is determined by the mechanism's architecture and the angular and translational displacement capabilities of each link. For parallel mechanisms, the workspace is limited by the need to avoid collision between the mechanical parts and the limits of the displacement in the joints. A detailed workspace

analysis is the most important step in the design of parallel mechanisms, and it has thus received attention from several researchers [9, 15].

The workspace analysis of the mechanism proposed here has been conducted by checking for the intersection of positions of the moving frame for all possible rotation angles. Considering the mechanism's kinematic constraints that the moving platform always exists in the fixed circular guide and that there is no interference between the circular Ps, the workspace is circular with a radius of  $r_a - r_b$ . This workspace is dexterous, which is defined to be the set of points that can be reached with any arbitrary orientation of the moving frame.

However, as discussed in Section 4, two types of singularities occur in the workspace, which is determined by the kinematic constraints when the radius of the moving platform ( $r_b$ ) is less than or equal to  $0.5r_a$ . These singularities lead to serious problems in the mechanism, such as breakage and loss of controllability. Thus, the singularity-free workspace must be selected and maximized for practical applications. This singularity-free workspace is circular and has a radius of  $r_s$ , which depends on  $r_b$ . The relationship between  $r_s$  and  $r_b$  is expressed as follows:

$$r_s = \begin{cases} r_b & (r_b \leq 0.5r_a) \\ r_a - r_b & (r_b > 0.5r_a) \end{cases} \tag{24}$$

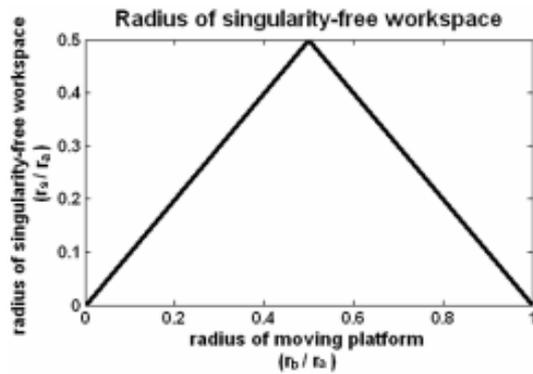


Fig. 5. Radius of the singularity-free workspace along the radius of moving platform ( $r_b/r_a$ ).

Fig. 5 shows the radius of the singularity-free workspace ( $r_s$ ) with respect to  $r_b$ , illustrating that the singularity-free workspace is maximized when the  $r_b$  is equal to  $0.5 r_a$ .

## 6. Application example of the platform: PCB depaneling machine

To separate the individual PCB from the bigger production panel, the PCB depaneling system consists of a feeder, a positioning device, and a bridge-cutting tool. Generally, the  $X$ - $Y$  stage and the milling cutter are used as the positioning device and cutting tool, respectively. However, there are three major drawbacks: heavy moving mass, excessive milling dust, and danger of the tool breaking.

Fig. 6 shows the suggested conceptual design of the PCB depaneling system. A punching tool is adopted as the cutting tool of a bridge, as the punching process can reduce the cutting time and the volume of milling dust. The working life of the tool can also be extended. The punching tool has drawbacks in that the die needs to cut the bridge, and the orientation of the punch should be aligned along with the bridge.

The proposed 3-DOF planar parallel mechanism is used as the positioning device of the PCB. As the parallel mechanism has an open space above and below the end-effector, the synchronized punch-die can be operated effectively. In addition, because the orientation of the punching tool is fixed, the high rotation capability can carry out bridge cutting in an arbitrary orientation. The proposed PCB depaneling system using the punching tool and parallel mechanism has the advantages of small moving mass, no milling dust, and long tool life.

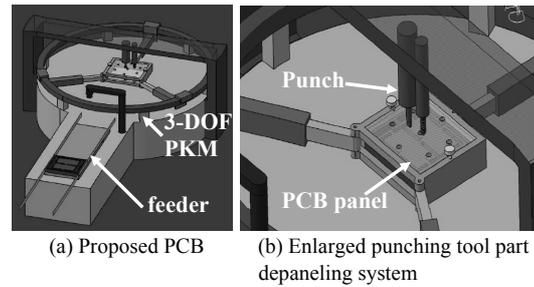


Fig. 6. Conceptual design of the PCB depaneling machine using the planar parallel mechanism and a punching tool.

## 7. Conclusion

This paper has presented a new planar parallel mechanism with one rotational and two translational DOF capable of a continuous 360-degree rotation of the platform. The kinematics, singularities, and workspace aspects have been analyzed to validate the reliability of the proposed mechanism as a positioning platform. The mechanism has been proven to have a 360-degree rotation in the  $0.5r_a$  radius of a circle-shaped singularity-free workspace. One applicable example for the PCB depaneling machine has also been proposed. The mechanism can be used in various fields where high rotational capability is required.

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