Proceedings of the 2002 IEEE
International Conference on Robotics & Automation
Washington, DC • May 2002

A New Three-Degree-of-Freedom Parallel Manipulator

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Abstract

In this paper, a novel spatial three degree-of-freedom, that are two degrees of translational freedom and one degree of orientational freedom, parallel manipulator is proposed. The parallel manipulator consists of a base plate, a movable platform, and three non-identical connecting legs. The kinematics problems and velocity equation of the new parallel manipulator are given. Three kinds of singularities are presented. The parallel manipulator has wide application in the fields of industrial robots, motion simulators, and parallel kinematics machines.

1 INTRODUCTION

In the past two decades, there have been considerable developments in the field of parallel mechanisms because they can be used as industrial robots [1], simulators [2], force/torque sensors [3], micromanipulators [4], and parallel kinematics machines [5]. Parallel manipulators have many advantages over serial manipulators in terms of high load/weight ratio, velocity, stiffness, precision, and inertia. The major drawback of the manipulators is their limited range of motion. The comparison of these two types of manipulators is discussed in more detail by Cox and Tesar [6].

From the 80's of the last century, some studies have led to the identification of several mechanical architectures with potential applications in manipulators. Hunt [7] presented a systematic study of parallel mechanisms and introduced many geometries applicable to robot arms. Other configurations for parallel manipulators with specified number and type of degrees of freedom have also been proposed. Behi [8] described a 6-DOF configuration with three legs where each leg consists of a PRPS chain. Hudgens and Tesar [9] investigated a device with six inextensible legs where each leg is driven by a four-bar mechanism mounted on the base.

There are also many 3-DOF parallel manipulators. Among these architectures, some (e.g. the 3-RPS parallel manipulator) have special kinematics characteristics [10]. Some of them (e.g. DELTA [11], Star like [12], the manipulator described in [13] and spherical 3-DOF parallel manipulators [14,15]) are with pure translational or orientational degrees of freedom. Some (e.g. planar 3-DOF parallel manipulator [16,17]) are planar parallel manipulators. It is necessary to design a spatial three degree-of-freedom parallel manipulator combining spatial translational and orientational degrees of freedom in the context of industrial applications, such as industrial robots, motion simulators, and parallel kinematics machines.

In this paper, a new spatial 3-DOF parallel manipulator with three non-identical chains is developed. The movable platform has three degrees of freedom, which are two degrees of translational freedom and one degree of orientational freedom, with respect to the base plate. The kinematics problems and velocity equation of the new parallel manipulator are given. Three kinds of singularities are presented. The parallel manipulator studied here has wide application in the fields of industrial robots, motion simulators, and parallel kinematics machines. The kinematics, velocity and singularity analyses presented in this paper can be of great help in the design, application and control of such devices.

2 DESCRIPTION OF THE MANIPULATOR

2.1 Manipulator Structure

Our manipulator, as shown in Fig.1, contains a triangular plate referred to as the moving platform (5). The platform is an isosceles triangle described by its parameter r , where $O'P_i = r$ ($i = 1, 2, 3$), as shown in Fig.2. The vertices of this platform are connected to a fixed-base plate, consisting of (1), (9) and (10), through three legs (3), (7) and (12). The legs

(3) and (7) have identical chains, each of which consists of a constant link, a planar four-bar parallelogram, which is connected to a passive revolute joint (4) or (6) at the top end and a prismatic joint (2) or (8) at the other. The third leg (12) consists of a constant link, also a planar four-bar parallelogram, which is connected to a revolute joint (13) at the top end and a cylinder joint (11), consisting of a revolute joint and a prismatic joint, at the other. Legs (3) and (7) are in a same plane, that is the axis of the revolute joint (4) or the prismatic joint (2) is identical with that of the joint (6) or (8). The axis of the cylinder joint (11) is parallel to that of the joint (2). Parameter a designate the distance the axis of the cylinder joint (11) being far away from that of the joint (2). The prismatic joint of each of the three legs are actuated. The movement of the moving platform is accomplished by the movement of three prismatic joints.

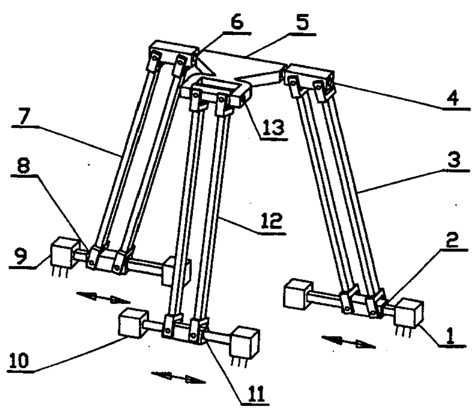


Fig.1 The New Spatial 3-DOF Parallel Manipulator

2.2 Manipulator Mobility

The first issue to address in the design of a mechanical system is to demonstrate its capability. In this case, the proposed manipulator is a general manipulation device that must have three degrees of freedom when the input elements are active.

Due to the arrangement of the links and joints, two legs (3) and (7) provide two constraints on the rotation of the moving platform about the x and z axes and the translation along $x-$axis. Two joints (11) and (13) for the third leg (12) have parallel axes as shown in Fig.1. The third leg can provide constraints on the rotation of the moving platform about x and z axes. Hence, the combination of the three legs constrains the rotation of the moving platform with respect to x and z axes and the translation along $x-$ axis. This leaves the mechanism with two translational degrees and one rotational degree of freedom.

The output of the manipulator is identical with that of the manipulator proposed in [16].

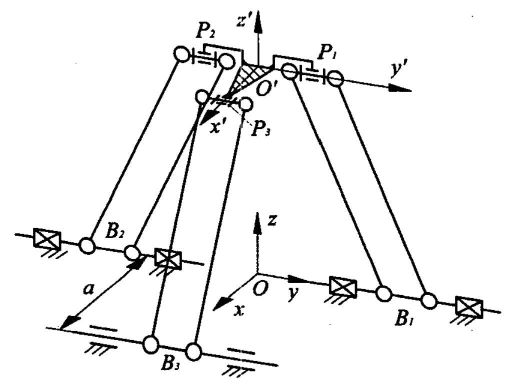


Fig.2 The Geometric Description of the 3-DOF Parallel Manipulator

3 KINEMATICS OF THE MANIPULATOR

Mechanism kinematics deals with the study of the mechanism motion as constrained by the geometry of the links. Typically, the study of mechanism kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics. The inverse kinematics problem involves mapping a known pose (position and orientation) of the moving platform of the mechanism to a set of input joint variables that will achieve that pose. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs. In General, as the number of closed kinematics loops in the parallel mechanism increases, the difficulty of solving the forward kinematics relationships increases. The inverse and forward kinematics problems of our parallel mechanism can be described in closed forms.

3.1 The Inverse Kinematics

A kinematics model of the manipulator is developed as shown in Fig.2. The vertices of the moving platform are denoted as platform joints P_i ($i=1,2,3$), and the end points of three legs connecting the base platform are denoted as B_i ($i=1,2,3$). A fixed reference system $\Re: O-xyz$ is attached to the base platform with $O-xy$ is on the plane defined by the first and second legs and the y-axis directed along B_2B_1. Another reference frame, called the top frame $\Re': O'-x'y'z'$, is located at the center of the side P_1P_2. The x'-axis is directed along $O'P_3$ and y'-axis directed along P_2P_1. The link length for each leg is denoted as L, where $P_iB_i = L$, $i=1,2,3$. What we should note that, in some case, the length of the link P_3B_3 can be different from that of P_1B_1 and P_2B_2.

The objective of the inverse kinematics solution is to define a mapping from the pose of the moving platform in a Cartesian space to the set of actuated inputs that achieve that pose. For this analysis, the pose of the moving platform is considered known, and the position is given by the position vector $[\boldsymbol{O}']_{\Re}$ and the orientation is given by a matrix $\boldsymbol{Q}$. And there are

$$[\boldsymbol{O}']_{\Re} = (0 \quad y \quad z)^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{Q} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \tag{2}$$

where the angle ϕ is the orientational degree of freedom of the moving platform with respect to y-axis. The coordinate of the point P_i in the frame $\Re'$ can be described by the vector $[\boldsymbol{p}_i]_{\Re'}$ ($i=1,2,3$), and

$$[\boldsymbol{p}_1]_{\Re'} = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix},\ [\boldsymbol{p}_2]_{\Re'} = \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix},\ [\boldsymbol{p}_3]_{\Re'} = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Vectors $[\boldsymbol{b}_i]_{\Re}$ ($i=1,2,3$) will be defined as the position vectors of base joints in frame $\Re$, and

$$[\boldsymbol{b}_1]_{\Re} = \begin{pmatrix} 0 \\ y_1 \\ 0 \end{pmatrix},\ [\boldsymbol{b}_2]_{\Re} = \begin{pmatrix} 0 \\ y_2 \\ 0 \end{pmatrix},\ [\boldsymbol{b}_3]_{\Re} = \begin{pmatrix} a \\ y_3 \\ 0 \end{pmatrix} \tag{4}$$

The vector $[\boldsymbol{p}_i]_{\Re}$ ($i=1,2,3$) in frame $O-xyz$ can be written as

$$[\boldsymbol{p}_i]_{\Re} = \boldsymbol{Q}[\boldsymbol{p}_i]_{\Re'} + [\boldsymbol{O}']_{\Re} \tag{5}$$

Then the inverse kinematics of the parallel manipulator can be solved by writing following constraint equation

$$\left\| [\boldsymbol{p}_i - \boldsymbol{b}_i]_{\Re} \right\| = L \qquad i=1,2,3 \tag{6}$$

Hence, for a given manipulator and for prescribed values of the position and orientation of the platform, the required actuator inputs can be directly computed from Eq. (7), that is

$$y_1 = \pm\sqrt{L^2 - z^2} + r + y \tag{7}$$

$$y_2 = \pm\sqrt{L^2 - z^2} + y - r \tag{8}$$

$$y_3 = \pm\sqrt{L^2 - (-r\sin\phi + z)^2 - (r\cos\phi - a)^2} + y \tag{9}$$

From Eqs.(7)~(9), we can see that there are eight inverse kinematics solutions for a given pose of the parallel manipulator.

3.2 The Direct Kinematics

The objective of the forward kinematics solution is to define a mapping from the known set of the actuated inputs to the unknown pose of the moving platform. For the architecture with prismatic actuators, as shown in Fig.2, the inputs that are considered known are y_1, y_2 and y_3. The unknown pose of the moving platform is described by the position vector $[\boldsymbol{O}']_{\Re}$ and the angle ϕ. From Eqs. (7) and (8), we can obtain

$$y = \frac{y_1 + y_2}{2} \tag{10}$$

Substituting Eq.(10) into Eq.(8) leads to

$$z = \pm\sqrt{L^2 - \left(r + \frac{y_2 - y_1}{2}\right)^2} \tag{11}$$

If z and y are obtained from Eqs. (11) and (10), the direct solutions of angle ϕ can be reached as

$$\phi = 2\tan^{-1}(t) \tag{12}$$

where

$$t = \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{C + B} \tag{13}$$

with

$$A = 2zr$$
$$B = 2ar$$
$$C = (y - y_3)^2 + a^2 + z^2 + r^2 - L^2$$

From Eqs. (10)~(13), we can see that for the given values of y_1, y_2 and y_3, there are two solutions for y and z, respectively, and four solutions for ϕ. Therefore, the solution of the direct kinematics of the architecture can reach four.

From above analysis, we can see that the solution of inverse kinematics for the spatial 3-DOF parallel manipulator can reach eight and the solution of direct kinematics can be four, and all the solutions are described in closed forms.

4 VELOCITY EQUATIONS

Equation (6) can be differentiated with respect to time to obtain the velocity equations. This leads to an equation of the form

$$A\dot{\rho} = B\dot{p} \tag{14}$$

where $\dot{p}$ is the vector of output velocities defined as

$$\dot{p} = [\dot{y},\ \dot{z},\ \dot{\phi}]^T \tag{15}$$

and $\dot{\rho}$ is the vector of input velocities defined as

$$\dot{\rho} = [\dot{y}_1\ \ \dot{y}_2\ \ \dot{y}_3]^T \tag{16}$$

Matrices A and B are the 3×3 Jacobian matrices of the manipulator and can be expressed as

$$A = \begin{bmatrix} r+y-y_1 & 0 & 0 \\ 0 & y-r-y_2 & 0 \\ 0 & 0 & y-y_3 \end{bmatrix} \tag{17}$$

$$B = \begin{bmatrix} r+y-y_1 & z & 0 \\ y-r-y_2 & z & 0 \\ y-y_3 & z-r\sin\phi & ar\sin\phi - zr\cos\phi \end{bmatrix} \tag{18}$$

5 SINGULARITY ANALYSIS

Because singularity leads to a loss of the controllability and degradation of the natural stiffness of manipulators, the analysis of parallel manipulators has drawn considerable attention. Based on the forward and inverse Jacobian matrix, three kinds of singularities of parallel manipulators can be obtained [17].

5.1 The First Kind of Singularity

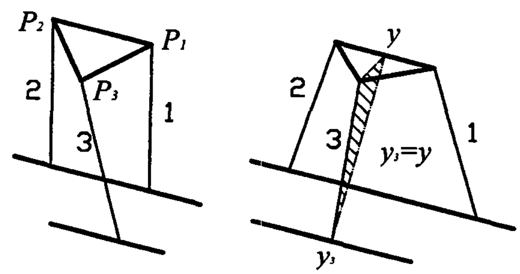
The first kind of singularity occurs when the following condition is satisfied:

$$\det(A) = 0 \tag{19}$$

This kind of singularity corresponds to the configuration that the moving platform reaches the limit of the workspace. This condition is encountered here when one of the diagonal entries of A vanishes, that is

$$(r+y-y_1)(y-r-y_2)(y-y_3) = 0 \tag{20}$$

This type of configuration is reached whenever one of the legs P_1B_1, P_2B_2 and P_3B_3 is in the plane paralleling to the $O-xz$ plane, as shown in Fig.3. For the first and second legs, this means that one of legs P_1B_1 and P_2B_2 is vertical to $y-$axis. If the first leg occurs, that is $r+y-y_1=0$, from Eqs.(7) and (8), we can see that $y-r-y_2=0$. Similarly, if the second leg occurs, the first leg must be in singular configuration.



(a) (b)

Fig.3 The First Kind of Singularity

(a) Legs 1 and 2 are vertical to the Base, (b) $y_3 = y$

5.2 The Second Kind of Singularity

The second kind of singularity occurs when we have following

$$\det(B) = 0 \tag{21}$$

which corresponds to the singularity being located inside the workspace of the manipulator. From Eqs.(21) and (18), we can obtain

$$zr(a\sin\phi - z\cos\phi)(2r + y_2 - y_1) = 0 \tag{22}$$

which leads to

$$zr = 0 \tag{23}$$

or

$$a\sin\phi - z\cos\phi = 0 \tag{24}$$

or

$$2r + y_2 - y_1 = 0 \tag{25}$$

- From Eq.(23), we can see that if $z=0$, the configuration of the manipulator is that the first and second legs are in the plane $O-xy$, as shown in Fig.4(a). When $r=0$, the manipulator is in architecture singularity.
- When the Eq.(24) is satisfied, one obtains

$$\tan\phi = z/a \tag{26}$$

This corresponds the configuration the third leg

P_3B_3 being in the plane $P_1P_2P_3$, as shown in Fig.4(b).

- If the Eq.(25) is verified, that means two legs P_1B_1 and P_2B_2 are vertical to the plane $O-xy$, simultaneously, as shown in Fig.3(a). The configuration is identical with that of one of the singular configurations of the first kind of singularity.

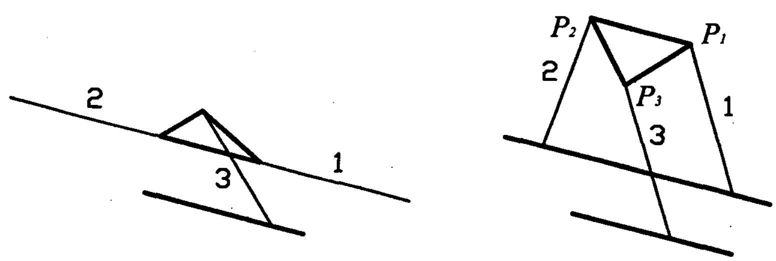
5.3 The Third Kind of Singularity

This kind of singularity occurs when both $\boldsymbol{A}$ and $\boldsymbol{B}$ become simultaneously singular. For the parallel manipulator studied here, the condition for which the singularity occurs should be: One of the first and second legs P_1B_1 and P_2B_2 is vertical to the plane $O-xy$.

Therefore, the 3-DOF manipulator of this paper is in singularity configuration when anyone of four following conditions occurs: *a)* one of three legs P_1B_1, P_2B_2 and P_3B_3 is in the plane paralleling to the $O-xz$ plane; *b)* the third leg P_3B_3 is in the plane $P_1P_2P_3$; *c)* $z=0$; *d)* $r=0$, which is the architecture singularity.

6 CONCLUSION

In this paper, a type of new spatial 3-DOF parallel manipulator with three non-identical chains is developed. The movable platform has three degrees of freedom, which are two degrees of translational freedom and one degree of orientational freedom, with respect to the base plate. And the inverse and direct kinematics problems for the manipulator are all described in closed-form. The results show that the solution of inverse kinematics can reach eight and the solution of direct kinematics can be four. The velocity equation of the new parallel manipulator is given. And three kinds of singularities are presented. The parallel manipulator has wide application in the fields of industrial robots, simulators, micromanipulators, and parallel kinematics machines.



(a) (b)

Fig.4 The Second Kind of Singularity

(a) Legs 1 and 2 are in the plane $O-xy$, (b) Legs 3 is in the plane $P_1P_2P_3$

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