



A calibration method of redundantly actuated parallel mechanism machines based on projection technique

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ABSTRACT

This paper presents a new calibration method for redundantly actuated parallel mechanism machines without measuring stiffness of actuating joints directly. The stiffness measurement of the actuating joints was a mandatory procedure to calibrate the redundantly actuated parallel mechanism in previous works. A new error propagation formula by using projection technique is established, which projects the constraint force terms onto the orthogonal complementary terms, in order to remove the need to know joint stiffness. Two sets of experimental verification are presented: (1) a two d.o.f. Gosselin's mechanism manipulator with three actuators and (2) a three d.o.f. parallel platform with four actuators.

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1. Introduction

Recently, the redundantly actuated parallel mechanism machine (RAPMM) have been focused on since the RAPMM can overcome the classical problems of the conventional parallel mechanism machine [1]. The problems include relatively small workspace, internal uncontrollable region that reduces the workspace even smaller, nonlinear control behaviour with respect to the Cartesian coordinate, and complex theories to design the parallel mechanism machine [2]. The RAPMM can compensate the singular region and enlarge the workspace with addition of extra actuators to the degrees of freedom of the given mechanism. The redundant actuation also enables the machine to have actuators with smaller power.

In case of calibration of the parallel mechanism manipulator, self-calibration strategies have come into being for one-step calibration [3–6]. The calibration of the RAPMM, however, gets to be more complex since the manipulator has excessive actuators to the degrees of freedom of the mechanism and would include internal forces during the position control. If there are the kinematic error in link length and encoder indexing error from the nominal model, the constraint force should make the machine encounter the undesired position and orientation where the configuration satisfies the force equilibrium. As a previous work, Jeong et al. analyzed error propagation mechanism for RAPMM and verified the calibration procedure on a two d.o.f. Gosselin's mechanism manipulator with three actuators [7].

The suggested calibration procedure in the study, however, required the measurement of the actuating joints. That is, the

stiffness of the actuating joints should be measured before the parallel mechanism machine is assembled. The stiffness measurement of each actuating joint is one of the important issues in the practical calibration process of the RAPMM. The measurement of each actuating joint is not easy for the general multi-degrees of freedom parallel mechanism machines such as Eclipse parallel machine series [1,8], but time consuming experiment should be accomplished. Moreover, the stiffness of the each joint could be changed after the assembly of the machine since the bearings and connecting components of the actuating joints might be preloaded after the manufacture of the machine. The even small change of the stiffness can affect the position of end-effector of the RAPMM, and the change should be additional error source of the RAPMM.

To overcome this drawback, this study proposes a new calibration procedure for RAPMM using projection theory [9]. According to the new error propagation theory, measurement of stiffness of actuating joints and external force for the calibration of RAPMM is not required, which was an essential process in the previous work. Thus, the calibration process becomes to be possible even for assembled commercial RAPMM without any stiffness information.

In Section 2 of this study, we briefly review the background theory about RAPMM calibration, and new calibration algorithm is formulated. In Section 3, two experimental results are suggested for two RAPMM: a two d.o.f. manipulator with three actuators and a three d.o.f. parallel platform with four actuators. Finally, we conclude in Section 4 with brief summary of the new calibration method.

2. Calibration method using projection technique

The calibration is a process finding the accurate kinematic parameters by applying the relationship of the joint values to the

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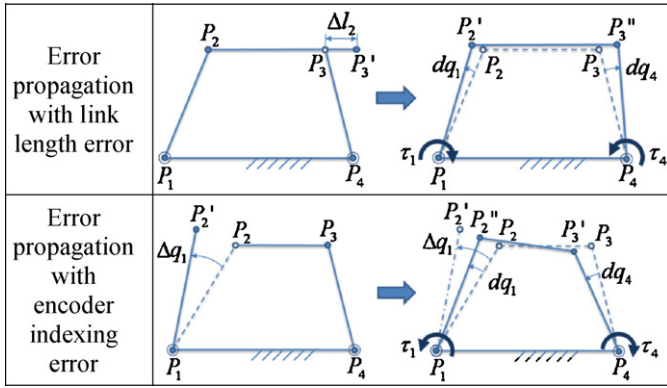


Fig. 1. Error propagation mechanism in case of redundant actuation.

end effector pose. In order to develop the calibration method for RAPMM one should consider not only the kinematic error sources but also the influence to the position by constraint force and the deflection of joints.

Fig. 1 depicts error propagation mechanism for redundant actuation in case of a four-bar linkage. Since the four-bar linkage has one degree of freedom, only one actuator is sufficient for operation. In the figure joint #1 and joint #4 are active in redundant case. In the case of redundant actuation, the actuator at joint #1 cannot keep the position of the mechanism because of the existence of another actuator at joint #4. These two actuators try to maintain the position of P_2' and P_3 respectively and this generates the conflict of driving forces, namely, constraint force.

The general error propagation mechanism of RAPMM can be modelled with kinematic parameters and position of the parallel machine. The detailed formulae were proposed in previous work [7]. The actual deviation vectors of independent joints dq_u and actuating joints dq_r can be depicted as (1) and (2), respectively.

$$dq_u = (\Gamma^T \mathbf{K} \Gamma)^{-1} \Gamma^T \mathbf{K} (\Delta q_r - \mathbf{H} \Delta \rho) - (\Gamma^T \mathbf{K} \Gamma)^{-1} P \quad (1)$$

$$dq_r = \Gamma dq_u + \mathbf{H} \Delta \rho \quad (2)$$

where Δq_r is encoder indexing error of actuating joints and $\Delta \rho$ is error of kinematic parameters. P is an external force vector including gravity force onto the parallel mechanism machine. Γ is a constraint Jacobian matrix which represents the position and configuration of the RAPMM. \mathbf{H} is an identification matrix which depicts the relationship between the positional deviation of independent joints and kinematic parameter error such as link length error. \mathbf{K} is a stiffness matrix of actuating joints.

The positional error of the tool plate of the parallel mechanism machine dP_t can be expressed as (3):

$$dP_t = \mathbf{J}_f dq_u + \mathbf{J}_\rho \Delta \rho \quad (3)$$

where \mathbf{J}_f is a forward Jacobian matrix, which represents the relationship between joint deviation and positional error of the tool platform. \mathbf{J}_ρ is an identification Jacobian matrix, which represents the relationship between kinematic parameter error joint deviation and the positional error of the tool platform. These matrices can be calculated from the command position of the machine.

In (1), the matrices Γ and \mathbf{H} can be calculated since the nominal position and orientation of the mechanism is given as the command position of the mechanism. Thus, the stiffness \mathbf{K} and external force P should be measured or calculated in order to obtain kinematic parameter Δq_r and $\Delta \rho$, which are calibration target. In the previous work [7], the \mathbf{K} and P should be measured before machine assembly.

In this study, in contrast, only position of a tool platform and the encoder following error are required to be measured with respect to the command position of the machine. The novel calibration

technique enables the calculation of the \mathbf{K} and P as part of the algorithm. The main contribution of this study is to propose a novel calibration algorithm which can eliminate the measurement procedure about the active joint stiffness.

As mentioned above, joint deflections of RAPMM are determined by encoder indexing error as well as joint stiffness and external force in (1). We introduce the new measurement value, that is, 'following error': difference between command position and actual position in actuators.

The following error dq_m can be defined as (4):

$$dq_m = dq_r - \Delta q_r = \Gamma dq_u + \mathbf{H} \Delta \rho - \Delta q_r \quad (4)$$

where Eqs. (3) and (4) are expressed together as following form (5):

$$\begin{bmatrix} dP_t \\ dq_m \end{bmatrix} = \begin{bmatrix} \mathbf{J}_f \\ \Gamma \end{bmatrix} dq_u + \begin{bmatrix} \mathbf{J}_\rho \\ \mathbf{H} \end{bmatrix} \Delta \rho - \begin{bmatrix} 0 \\ \Delta q_r \end{bmatrix} \quad (5)$$

Then, the first matrix term of right-hand side of (5) can be defined as matrix \mathbf{Q} such as (6), which is the beginning of identification process of new method.

$$\mathbf{Q} \equiv \begin{bmatrix} \mathbf{J}_f \\ \Gamma \end{bmatrix} \quad (6)$$

where \mathbf{Q} is $(n+l) \times n$ dimensional rectangular matrix whose rank is n . It is obvious that rank of \mathbf{Q} is n . Then matrix \mathbf{Q} satisfies the following relationship (7):

$$\mathbf{R} \mathbf{Q} = \mathbf{0} \quad (7)$$

where $\mathbf{R} \equiv [\mathbf{I} - \mathbf{Q}(\mathbf{Q}^T \mathbf{Q})^{-1} \mathbf{Q}^T]$

The matrix \mathbf{R} is the projection matrix which project specific vector into zero space, that is, its orthogonal complement. Multiplying (5) by projection matrix \mathbf{R} , (5) is simplified as

$$\mathbf{R} \begin{bmatrix} dP_t \\ dq_m \end{bmatrix} = \mathbf{R} \begin{bmatrix} \mathbf{J}_\rho \\ \mathbf{H} \end{bmatrix} \Delta \rho - \mathbf{R} \begin{bmatrix} 0 \\ \Delta q_r \end{bmatrix} \quad (8)$$

Here, we can check that vector $\mathbf{Q} dq_u$ is projected into its orthogonal complement by projection matrix \mathbf{R} and is eliminated from Eq. (8). This has very important meaning since dq_u vanishes due to the projection. In (5), dP_t and dq_m will be given by measurement, and $\Delta \rho$ and Δq_r are kinematic parameters to be identified. The dq_u is unknown value because of the absence of information about joint stiffness and external force. However, Eq. (8) does not contain dq_u term by projection, so that encoder indexing error and external force do not need to be considered in the identification process.

After simplifying (8), we can acquire (9).

$$\mathbf{R} \begin{bmatrix} dP_t \\ dq_m \end{bmatrix} = \begin{bmatrix} \mathbf{R} \begin{bmatrix} \mathbf{J}_\rho \\ \mathbf{H} \end{bmatrix} & -\mathbf{R}_2 \end{bmatrix} \begin{bmatrix} \Delta \rho \\ \Delta q_r \end{bmatrix} \quad (9)$$

where $\mathbf{R} = [\mathbf{R}_1 \quad \mathbf{R}_2]$, $\mathbf{R}_1 \in \mathfrak{R}^{(n+l) \times n}$, $\mathbf{R}_2 \in \mathfrak{R}^{(n+l) \times l}$

Using this relationship, an objective function for the calibration can be set up as (10)

$$\min_{\Delta q_r, \Delta \rho} \sum_{i=1}^N \|e_i\|^2 = \min_{\Delta q_r, \Delta \rho} \sum_{i=1}^N \left\| \mathbf{R} \begin{bmatrix} d\hat{P}_t \\ d\hat{q}_m \end{bmatrix} - \begin{bmatrix} \mathbf{R} \begin{bmatrix} \mathbf{J}_\rho \\ \mathbf{H} \end{bmatrix} & -\mathbf{R}_2 \end{bmatrix} \begin{bmatrix} \Delta \rho \\ \Delta q_r \end{bmatrix} \right\|^2 \quad (10)$$

where $d\hat{P}_t$ is measured displacement of end-effector and $d\hat{q}_m$ is measured following error of actuating joints. In this way, kinematic parameters and encoder indexing error can be identified directly if we obtain actual end-effector pose and following error of actuating joint.

3. Experimental results

3.1. Case study 1: Two d.o.f. mechanism machine with three actuators

The first RAPMM to be calibrated is a two d.o.f. mechanism machine with three actuators [9]. In Fig. 2, the kinematic error modelling of the machine is depicted. In the figure, B_1 , B_2 , and B_3

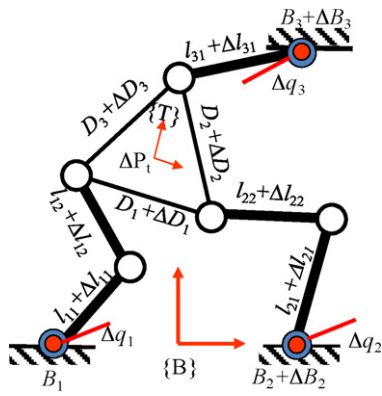


Fig. 2. Kinematic error modelling for the two d.o.f. parallel mechanism machine.

represent the position of the actuating joints where three servo motors are located, respectively. P_t is the position of the end-effector, which is the center of a triangular tool plate. The reference coordinate for the measurement is set up as follows: Z-axis of the reference frame is set to be a normal vector to the tool plate. X-axis is set to be a vector which connects between B_1 and B_2 . The position of B_1 is set to be $(-300, 0)$ point.

The end-effector position is measured by Leica™ AT901-MR laser tracker system, whose measurement uncertainty is $\pm 10 \mu\text{m} + 5 \mu\text{m}/\text{m}$ in $2.5 \text{ m} \times 5 \text{ m} \times 10 \text{ m}$ volume [11]. The following errors are measured by the encoders built in three servo motors. The measurement system is depicted in Fig. 3.

Fig. 4 shows the measured point set and sequence in the machine's workspace. We repeated the forward and backward path for three times, and identified the error parameters using the first 30 measured position datum among 38 points. Last 8 points are utilized for the verification, which are not considered in optimization.

The calibrated error parameters are shown in Table 1. The actual kinematic parameters are applied to the inverse kinematics

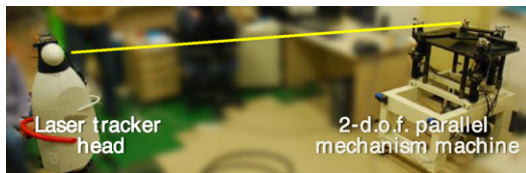


Fig. 3. Position measurement of the tool platform of 2-d.o.f. machine by a laser tracker system.

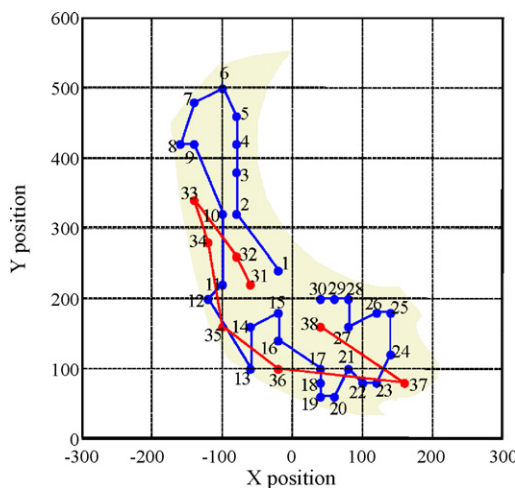


Fig. 4. Measurement position for the calibration and verification of the two d.o.f. parallel mechanism machine.

Table 1

Identified kinematic parameters of the 2-d.o.f. parallel machine with three actuators.

Nominal value		Identified error		Actual value	
l_{11}	280 mm	Δl_{11}	0.089 mm	$l_{11} + \Delta l_{11}$	280.089 mm
l_{12}	280 mm	Δl_{12}	-0.244 mm	$l_{12} + \Delta l_{12}$	279.756 mm
l_{21}	280 mm	Δl_{21}	0.024 mm	$l_{21} + \Delta l_{21}$	280.024 mm
l_{22}	280 mm	Δl_{22}	-0.458 mm	$l_{22} + \Delta l_{22}$	279.542 mm
l_{31}	280 mm	Δl_{31}	-0.177 mm	$l_{31} + \Delta l_{31}$	279.823 mm
B_{2x}	300 mm	ΔB_{2x}	-0.547 mm	$B_{2x} + \Delta B_{2x}$	299.453 mm
B_{3x}	150 mm	ΔB_{3x}	-0.076 mm	$B_{3x} + \Delta B_{3x}$	149.924 mm
B_{3y}	420 mm	ΔB_{3y}	-0.037 mm	$B_{3y} + \Delta B_{3y}$	419.963 mm
D_1	215 mm	ΔD_1	-0.042 mm	$D_1 + \Delta D_1$	214.958 mm
D_2	215 mm	ΔD_2	-0.029 mm	$D_2 + \Delta D_2$	214.971 mm
D_3	215 mm	ΔD_3	0.080 mm	$D_3 + \Delta D_3$	215.080 mm
P_{tx}	0 mm	ΔP_{tx}	0.067 mm	$P_{tx} + \Delta P_{tx}$	0.067 mm
P_{ty}	0 mm	ΔP_{ty}	-0.103 mm	$P_{ty} + \Delta P_{ty}$	-0.103 mm
		Δq_1	0.024°		
		Δq_2	0.064°		
		Δq_3	-0.057°		

in the controller of the machine. Then the additional measurement test is carried out again according to given point set for the validation of the improvement of positional accuracy. Fig. 5 presents the positional distance error before and after calibration. The symbol “○” and “×” marks mean the error in the forward direction and backward direction before calibration, respectively. The symbol “◇” and “+” marks mean the error in the forward direction and backward direction after calibration, respectively. After the calibration, the accuracy of the end effector is improved by 78.4% on the average. The maximum distance error is reduced from 0.740 mm to 0.141 mm.

3.2. Case study 2: Three d.o.f. planar platform with four actuators

A three d.o.f. planar platform with four actuators was selected for the second validation experiment, which mechanism and error modelling are depicted in Fig. 6. The machine consists of four actuating joints moving along two parallel guide ways, and a tool plate and two legs that connect the tool plate to the revolute joints C_1 and C_2 are installed on the guide-way. The detail kinematic equation and constraint equation of the machine are

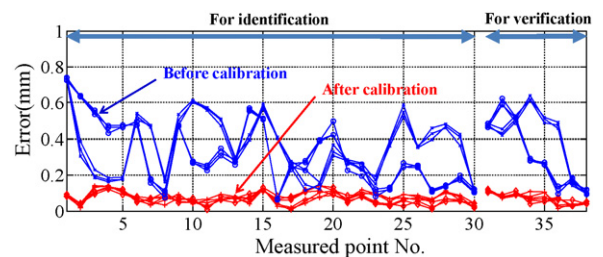


Fig. 5. Position error of two d.o.f. parallel mechanism machine before and after calibration.

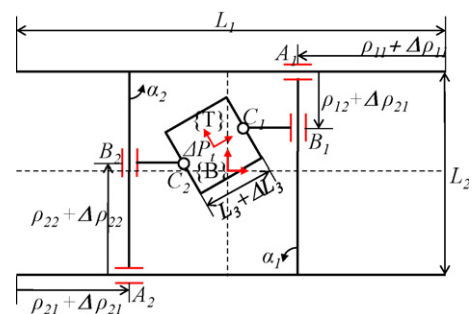


Fig. 6. Kinematic error modelling for the three d.o.f. parallel mechanism machine.

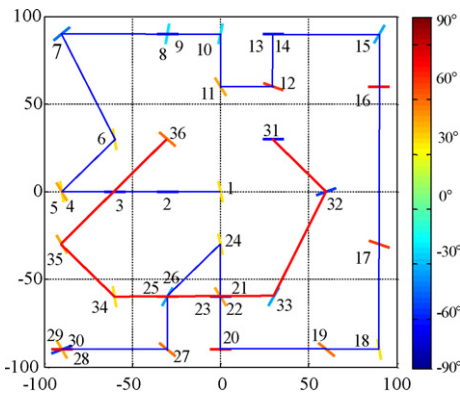


Fig. 7. Measurement position for the calibration of the three d.o.f. parallel mechanism machine.

Table 2

Identified kinematic parameters of the 3-d.o.f. parallel machine with four actuators.

Nominal value	Identified error	Actual value
L_3	230 mm	ΔL_3 0.079 mm
P_{rx}	0 mm	ΔP_{rx} -0.111 mm
P_{ry}	0 mm	ΔP_{ry} -0.064 mm
		$L_3 + \Delta L_3$ 230.079 mm
		$P_{rx} + \Delta P_{rx}$ -0.111 mm
		$P_{ry} + \Delta P_{ry}$ -0.064 mm
	$\Delta \rho_{11}$	-0.151 mm
	$\Delta \rho_{12}$	-0.421 mm
	$\Delta \rho_{21}$	-0.182 mm
	$\Delta \rho_{22}$	0.136 mm
	α_1	-0.065°
	α_2	0.035°

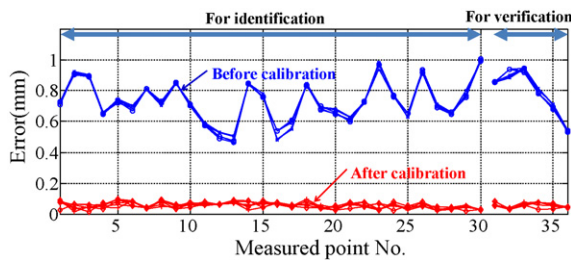


Fig. 8. Position error of three d.o.f. parallel mechanism machine before and after calibration.

presented in [10]. We assumed the two guide-ways which A_1 and A_2 move along are perfectly parallel since they share two different rails.

The nine error parameters are selected to guarantee the identifiability. The end-effector position is measured by Leica™ AT901-MR laser tracker and the following errors are measured by the encoders of 4 linear motors. Fig. 7 shows the measured point set and sequence in machine's workspace. Among 36 measuring points, we identified the error parameters using first 30 points. Last 6 points are used for the verification which is not considered in optimization.

The optimized error parameter values are shown in Table 2. The result is applied to the kinematics and we measured the end

effector position again according to given point set for the validation of the improvement of positional accuracy. Fig. 8 presents the positional distance error before and after calibration. After the calibration, the positional accuracy of the end effector is improved by 92.4% on the average. The maximum distance error is reduced from 1.006 mm to 0.097 mm.

4. Conclusion

In this study, we presented a novel calibration method for redundantly actuated parallel mechanism machines. The identification process was established using projection technique which makes the influence of joint stiffness and external force to be ignored. That is, one can identify the error parameters of RAPMM without any information on the joint stiffness. The proposed method was validated by two sets of experiments. After the calibration, the positional accuracy was improved by 78.4% for the 2-d.o.f. parallel machine and 92.4% for the 3-d.o.f. parallel machine, respectively.

Meanwhile, we proceed to the research on the prediction of the stiffness values of active joints using proposed method. The ratio of stiffness values can be found out in the parallel machine with one actuation redundancy.

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