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Analysis of a new planar 3-DOF parallel manipulator with two PPR chains

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Abstract— *In this paper, a new type of planar three-degree-of-freedom parallel mechanism is presented. This has been designed in order to obtain a larger dexterous workspace than a conventional parallel mechanism and a symmetric and compact structure. This mechanism employs two identical PPR chains to support the moving platform. Since the first P, which denotes sliding motion, and the second one are perpendicular to each other, this mechanism has a platform structure with decoupled-motion architecture. From this feature, this mechanism has simple kinematics and large workspace. In this paper, both inverse and forward kinematics are studied. The workspace and singularity analysis are presented and the redundant actuation is proposed as a solution for singularity problem. Furthermore the application of this mechanism with punching system is presented.*

Keywords: parallel mechanism, kinematics, workspace, singularity, 3-DOF.

I. Introduction

The production of electronic machine usually requires a depaneling process for separating individual printed circuit boards from the bigger production panel, which is needed in electronic mass production to provide more capacity from programming processes. The process quality can be evaluated based on high speed, high accuracy, low cost and less generating dust. Current depaneling process is milling process, which can generate much dust and has a limitation of increasing speed due to breakage of tools. To enhance the capacity, a new depaneling system, which includes punching system and part handling system, is proposed. A part handling system, of which the motion is on a plane with three degrees of freedom enabling to tilt ± 90 degrees, requires fast movement, high accuracy, and large workspace. Although the serial manipulators may have three-degrees-of-freedom required by the part handling in depaneling process, it is difficult to realize high angular velocity and acceleration of the moving platform by eccentric actuation, which is required by the punching system. Parallel mechanism has several advantages, which include high velocity, lower inertia, higher accuracy and stiffness, over the serial ones, which make them more suitable for part handling in this new depaneling process. Due to such advantages, several researchers and industries have shown interest in parallel mechanisms [1], [2], [3]. However, they have drawbacks such as complicated kinematics, limited workspace, and coupled-motion characteristics.

A planar three-degree-of-freedom parallel mechanism that is currently considered has the same characteristics as that shown above. Several researchers have already analyzed and built real machines [4], [5], [6]. Most planar three-degree-of-freedom parallel mechanism are composed of three serial chains that each has three joints. Although some of them have high rotational ability, the machine size would be extremely large if the rotational ability satisfied all the desired workspace [7], [8].

In this paper, a new type of planar three-degree-of–freedom parallel mechanism is proposed. This mechanism has large dexterous workspace having large tilting angle, ± 90° and a compact structure. Moreover, its translational motions are always driven by certain active joints so that it can be considered as decoupled motion architecture. Considering this fact, the closed form solutions of the kinematic problem are addressed. Jacobian matrix is given and singularities are analyzed. The descriptions of two types of workspace are provided. From these results, the application of depaneling system with punching system is proposed.

II. Kinematic Analysis

This section describes the kinematic design of the parallel mechanism, followed by a procedure for obtaining the closed-form solution of the inverse kinematics and forward kinematics.

A. Description of the mechanism

Fig. 1 illustrates the mechanism proposed in this paper. Fig. 2 shows the schematic of the mechanism. As shown in Fig. 2, this mechanism consists of two PPR serial subchains (P and R here denote prismatic and revolute joints), with the first P joint denoting sliding motion along the fixed linear guideway and putting on the guideway of the second P joint. The first P joint is expressed as vertical P joint and the second is horizontal P joint. The mechanism has three kinematic degrees of freedom, with the principal actuated joints indicated by arrows.

As illustrated in Fig. 2, a global reference frame $\Re : O - XY$ is located at the center of the two guideways and a moving frame $\Re' : o - xy$ is attached to the moving platform. The points A_i $(i = 1,2)$ denote the intersection of

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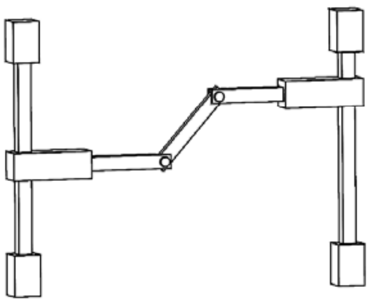


Fig. 1. A new planar 3-DOF parallel mechanism

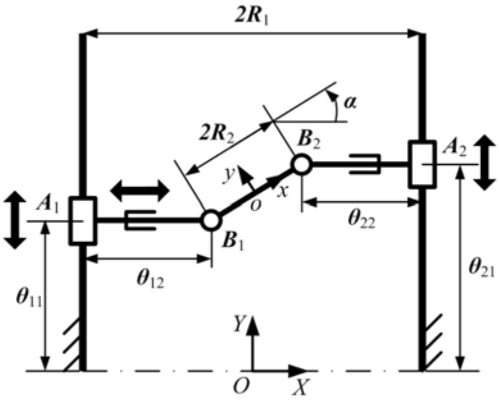


Fig. 2. Geometric parameters of the parallel mechanism

two prismatic joints that are located perpendicularly. Points B_i $(i=1,2)$ denote the revolute joints that are located between moving platform and horizontal prismatic joints. The geometric parameters of the mechanism are the length and the distance between two guideways $2R_1$ and the length of moving platform $2R_2$.

B. Inverse Kinematic Analysis

The inverse kinematics problem is concerned with determining the values of the actuated joints from the position and orientation of the moving frame attached to the moving platform. The position of the origin of moving frame o in the global reference frame can be described by the position vector $\boldsymbol{p}$, and the orientation of moving frame is given by a matrix $\mathbf{R}$. In addition, there are

$$\boldsymbol{p}=(x \quad y)^{\mathrm{T}} \tag{1}$$

$$\mathbf{R}=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix} \tag{2}$$

where the α is the angle of the moving frame with respect to the global reference frame.

In the global reference frame, the position vectors $\boldsymbol{b_i}$ of points B_i $(i=1,2)$ can be calculated as

$$\boldsymbol{b_i}=\mathbf{R}\boldsymbol{b}_{i\Re'}+\boldsymbol{p} \tag{3}$$

where $\boldsymbol{b}_{i\Re'}$ is the position vectors in the moving frame. This can be also expressed as

$$\boldsymbol{b_1}=(x-R_2\cos\alpha \quad y-R_2\sin\alpha)^{\mathrm{T}}$$

$$\boldsymbol{b_2}=(x+R_2\cos\alpha \quad y+R_2\sin\alpha)^{\mathrm{T}} \tag{4}$$

Furthermore, the vectors $\boldsymbol{a_i}$ of points A_i $(i=1,2)$ can be calculated by the fact that two prismatic joints are always perpendicular and these points are always on the vertical guideway.

$$\boldsymbol{a_1}=(-R_1 \quad y-R_2\sin\alpha)^{\mathrm{T}}$$

$$\boldsymbol{a_2}=(R_1 \quad y+R_2\sin\alpha)^{\mathrm{T}} \tag{5}$$

From $\boldsymbol{a_i}$ and $\boldsymbol{b_i}$, all input joint values can be obtained as follows

$$\theta_{11}=a_{1\mathrm{y}}=y-R_2\sin\alpha$$

$$\theta_{21}=a_{2\mathrm{y}}=y+R_2\sin\alpha$$

$$\theta_{12}=b_{1\mathrm{x}}-a_{1\mathrm{x}}=x-R_2\cos\alpha+R_1$$

$$\theta_{22}=b_{2\mathrm{x}}-a_{2\mathrm{x}}=x+R_2\cos\alpha-R_1 \tag{6}$$

where θ_{i1} $(i=1,2)$ are the vertical prismatic joint values and θ_{i2} $(i=1,2)$ are the horizontal prismatic joint values. From Eq. (6), one can know that there are four solutions by the sign of θ_{i2} $(i=1,2)$. For positive, B_i is located on the right side of A_i. In this paper, we are concerned about θ_{12} positive and θ_{22} negative that can eliminate the interference of moving platform with guideways.

C. Forward Kinematic Analysis

The forward kinematic problem is concerned with determining the position and orientation of the moving frame given the values for the actuated joints. In general the forward kinematics solution for most parallel mechanism is not unique, and it is quite difficult to solve the forward kinematics analytically. However, the forward

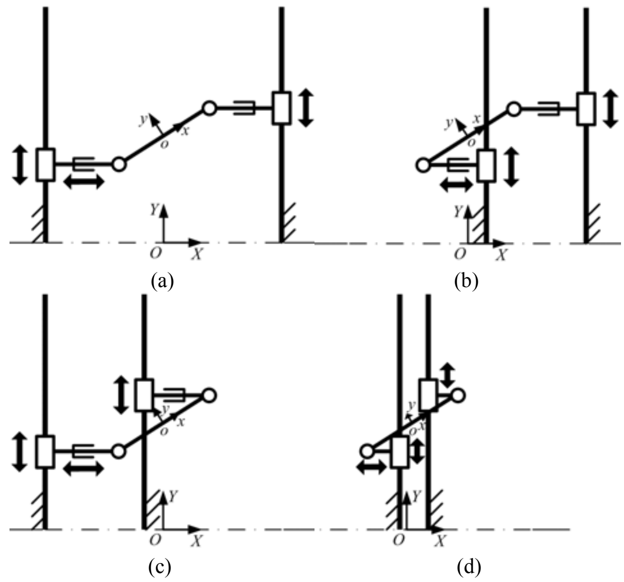


Fig. 3. Four solutions of inverse kinematics problem (θ_{12}, θ_{22})
(a) (+, -) (b) (-, -) (c) (+, +) (d) (-, +)

kinematics problem of our parallel mechanism is easy and can be described in closed form.

As the distance between the revolute joints of moving platform is required to be fixed, kinematic constraint equation is defined as follows:

$$\|\boldsymbol{b}_1 - \boldsymbol{b}_2\| = 2R_2 \quad (7)$$

in another form

$$(2R_1 + \theta_{22} - \theta_{12})^2 + (\theta_{21} - \theta_{11})^2 = 4R_2^{\,2} \quad (8)$$

From Eq. (8) and given active joint values, the passive joint value θ_{22} can be obtained

$$\theta_{22} = \theta_{12} - 2R_1 \pm \sqrt{4R_2^{\,2} - (\theta_{21} - \theta_{11})^2} \quad (9)$$

From the obtained values of active and passive joints, the vector $\boldsymbol{p}$ and rotation matrix $\mathbf{R}$ representing the position and orientation of the moving frame can be calculated as follows

$$\boldsymbol{p} = \frac{1}{2}(\boldsymbol{b}_1 + \boldsymbol{b}_2) = \begin{pmatrix} \frac{\theta_{12} + \theta_{22}}{2} & \frac{\theta_{11} + \theta_{21}}{2} \end{pmatrix}^{\mathrm{T}} \quad (10)$$

$$\mathbf{R} = \begin{bmatrix} \boldsymbol{R}_x & \boldsymbol{R}_y \end{bmatrix} \quad (11)$$

$$\boldsymbol{R}_x = \frac{1}{2R_2}(\boldsymbol{b}_2 - \boldsymbol{b}_1) = \frac{1}{2R_2}\begin{pmatrix} 2R_1 + \theta_{22} - \theta_{12} & \theta_{21} - \theta_{11} \end{pmatrix}^{\mathrm{T}}$$

$$\boldsymbol{R}_y = \frac{1}{2R_2}\begin{pmatrix} \theta_{11} - \theta_{21} & 2R_1 + \theta_{22} - \theta_{12} \end{pmatrix}^{\mathrm{T}} \quad (12)$$

From Eq. (9), there are two solutions for forward kinematic problem of this mechanism. In this paper the '+' case, which is a sign of square root, is chosen. From the above equations, the solutions of the forward kinematics can be obtained directly.

III. Jacobian Matrix

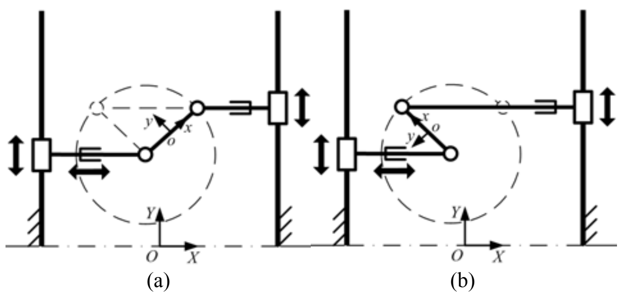
From Eq. (10) to (12), one can obtain the relationship between the input and output velocities by differentiating them with respect to time as follows:

$$\mathbf{A}\dot{\boldsymbol{x}} + \mathbf{B}\dot{\theta} = \boldsymbol{0} \quad (13)$$

where $\dot{\boldsymbol{x}}$ is the vector of output velocities and $\dot{\theta}$ is of input velocities as follows

$$\dot{\boldsymbol{x}} = \begin{pmatrix} \dot{x} & \dot{y} & \dot{\alpha} \end{pmatrix}^{\mathrm{T}}$$

$$\dot{\theta} = \begin{pmatrix} \dot{\theta}_{11} & \dot{\theta}_{12} & \dot{\theta}_{21} \end{pmatrix}^{\mathrm{T}} \quad (14)$$



(a) (b)

Fig. 4. Two cases of forward kinematics solution
(a) '+' case, (b) '-' case

The $\dot{\alpha}$ can be obtained from the characteristic of rotation matrix $\mathbf{R}$

$$\dot{\alpha} = \boldsymbol{R}_y^{\,\mathrm{T}} \dot{\boldsymbol{R}}_x \quad (15)$$

By differentiating Eq. (8) with respect to time, one can get the relationship between the passive joint velocity $\dot{\theta}_{22}$ and other active input joint velocities.

The matrices $\mathbf{A}$ and $\mathbf{B}$ are expressed as follows:

$$\mathbf{A} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \quad (16)$$

$$\mathbf{B} = \begin{bmatrix} -\frac{b}{2} & -a & \frac{b}{2} \\ -\frac{a}{2} & 0 & -\frac{a}{2} \\ \frac{a^2 + b^2}{4R_2^{\,2}} & 0 & -\frac{a^2 + b^2}{4R_2^{\,2}} \end{bmatrix} \quad (17)$$

where $a = 2R_1 + \theta_{22} - \theta_{12}$, $b = \theta_{21} - \theta_{11}$

The Jacobian matrix can be expressed as

$$\mathbf{J} = -\mathbf{A}^{-1}\mathbf{B} \quad (18)$$

IV. Singularity Analysis

Singularities are one of the most significant and critical problems in the design and control of parallel mechanisms because they lead to loss of the controllability, an instantaneous change of the mechanism's degree of freedom and degradation of the stiffness of the mechanism.

Singularities can be classified into three types. These are determined by calculating the velocity equation of the mechanism [9].

The first kind of singularity occurs when $\mathbf{B}$ is singular. This condition can be expressed as follows:

$$\det(\mathbf{B}) = 0 \quad (19)$$

In this case, the moving platform in which the moving frame is attached loses a degree of freedom.

The second kind of singularity occurs when $\mathbf{A}$ is singular, and this condition is

$$\det(\mathbf{A}) = 0 \quad (20)$$

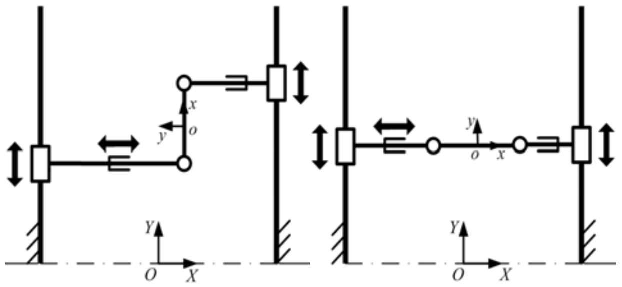
This means the moving platform is locally movable even when all the actuated joints are locked.

The third kind of singularity occurs when both $\mathbf{A}$ and $\mathbf{B}$ are singular.

$$\det(\mathbf{A}) = \det(\mathbf{B}) = 0 \quad (21)$$

In this case, the mechanism can undergo finite motions even when its actuators are locked, or some motions of the inputs cannot produce motion at the outputs.

From Eq. (16), one can obtain $a = 0$ to satisfy the singular condition of $\mathbf{A}$. This means $2R_1 = \theta_{12} - \theta_{22}$. Its physical meaning is that the line of two revolute joints on the moving platform is parallel to the y-axis as shown in Fig. 5 (a).



(a) (b)

Fig. 5. (a) singular case (moving platform is parallel to guideways), (b) non-singular case (moving platform is perpendicular to guideways)

From Eq. (17), one can obtain $a = 0$ or $a^2 + b^2 = 0$ to satisfy the singular condition of $\mathbf{B}$. For the case of $a = 0$, the meaning of $a = 0$ is already explained above. To satisfy the condition of $a^2 + b^2 = 0$, both variables have to be zero, $a = b = 0$. The $b = 0$ means $\theta_{11} = \theta_{21}$ and it means that the line of two revolute joints on the moving platform is parallel to the x-axis as shown in Fig. 5 (b). For both a and b to be zero, two revolute joints have to be in the same position. In other words, the length between revolute joints, $2R_2$ have to be zero. It means moving platform doesn't exist, and so that $a = 0$ is the only case to satisfy the singular condition of $\mathbf{B}$.

From the above analysis, all kinds of singularity occur if $a = 0$. So the only singularity of the mechanism occurs when the line of two revolute joints on the moving platform is parallel to the y-axis.

V. Workspace Analysis

The workspace of a mechanism refers to the set of all positions and orientations achievable by the moving frame. It is determined by the kind of mechanical architecture and the angular and translational displacement of each link. In the case of parallel manipulator, workspace is limited by the avoidance of collision between mechanical parts, and the limitation of displacement of the joints. So workspace analysis is the most important step in parallel mechanism design process, and so several researchers have conducted studies on the workspace analysis for parallel mechanisms [7], [8].

From the results of kinematic analysis above, one can calculate the workspace geometrically if the orientation α is specified. The orientation α has a range from -90 to +90 degrees to be the '+' type solution in forward kinematics problem as stated above. From Eq. (6) and the joint range of θ_{ij} $(i, j = 1,2)$, one can calculate the workspace that is enveloping surface of four straight lines. To avoid the interference between moving platform and guideways, the joint ranges are set to as follows:

$$\theta_{i1} \in [0, \quad 2R_1], \ (i = 1,2)$$
$$\theta_{12} \in [0, \quad 2R_1], \ \theta_{22} \in [-2R_1 \quad 0] \qquad (22)$$

The workspace with given orientation for some angle can be shown in Fig. 6 and for orientations ranging from –90 to +90 degrees in Fig. 7.

As shown in Fig. 7, the volume of workspace of this mechanism is changed by the orientation α. From this result, one can classify the workspace into two types, the reachable and dexterous workspaces [7]. This classification is well known in the robotics area. From the classification above, the reachable workspace is shown in Fig. 8 (a) and the dexterous in Fig 8 (b). In our case, we want to calculate the set of points that can be reached any arbitrary orientation of moving frame so the dexterous workspace is more meaningful.

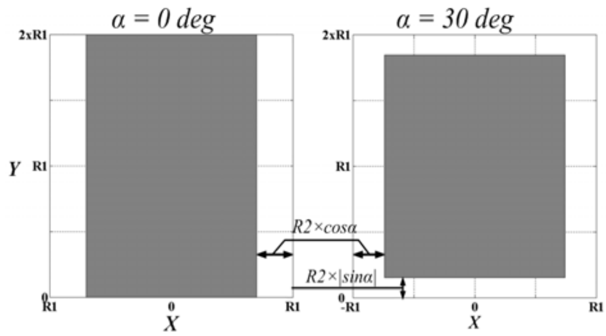


Fig. 6. Workspace for given orientation for some cases

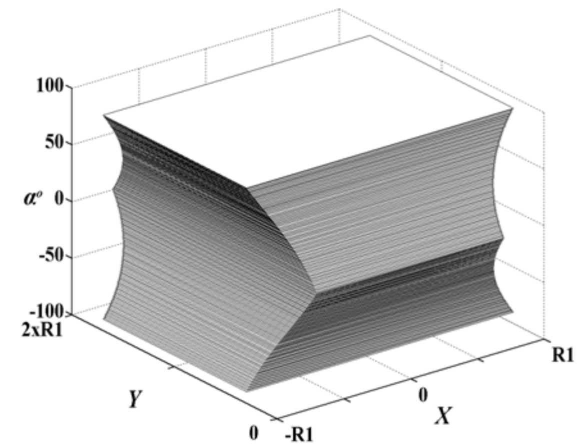
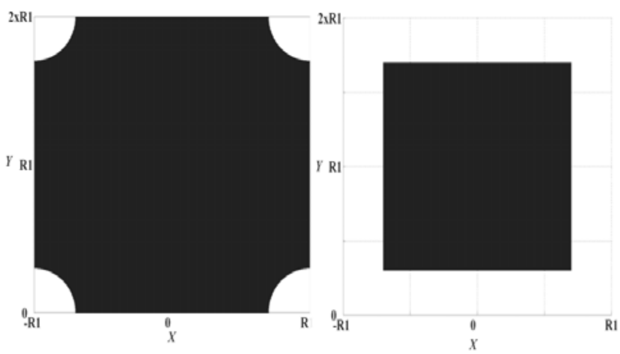


Fig. 7. Workspace for orientations ranging from –90 to +90 degrees



(a) (b)

Fig. 8. (a) Reachable workspace (b) Dexterous workspace

VI. Application

As described in Section I, this mechanism is developed to enhance the capacity of the depaneling system. To realize a new depaneling system, the mechanism has to realize planar three-degrees-of-freedom motion enabling to tilt –90 to +90 degrees. As shown in Section IV, the singularity occurs when the moving platform rotates ±90 degrees. To avoid the first and third type of singularities to be mentioned in Section IV, motion planning, changing mechanism or adding more actuators can be a solution. To avoid the second type of singularity, adding more actuators would be the solution. Despite of the problems redundant actuation introduces such as difficulties in computing the dynamics, complexities and the cost in control it is still attractive, especially in this mechanism, where the most important factor is to enlarge the dexterous workspace and increase the velocity and acceleration. That is, redundant actuation helps to eliminate singularities, increase velocity and acceleration and also stiffness [10], [11]. In addition it also provides a symmetric structure in this mechanism, which is a great advantage in designing. For these reasons it is reasonable to add a redundant actuator in this mechanism, so an additional actuator is added on the passive prismatic joint.

The conceptual design for a new depaneling system that is composed of punching system having a role to move vertically attached on the gantry, and parallel mechanism that is proposed in this paper can be shown in Fig 10. The shape of the moving platform is shown in Fig 9 is changed in this system compared with Fig 1. Although the moving platform shape is changed, the kinematics results are not changed.

The embodiment design based on the conceptual design is in progress, and we are going to make a prototype of the new depaneling system. The requirement of the production time for the newly developed depaneling system is less than 3 seconds per one PCB board which has 10 supporting materials to be removed. This is since the fastest tact time of the existing depaneling system, which is based on milling process and fixed part handling system, is about 5 seconds. To realize this production time, the moving platform has to satisfy both 450 deg/sec for maximum angular speed and 120 m/min for linear speed.

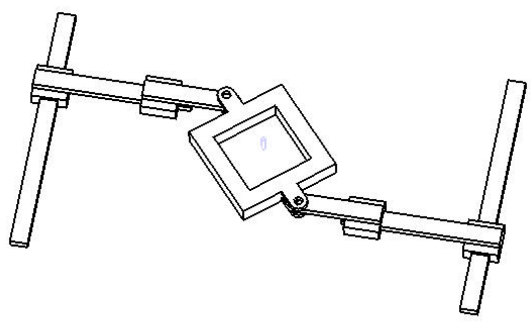


Fig. 9. Schematic with different moving platform for depaneling system

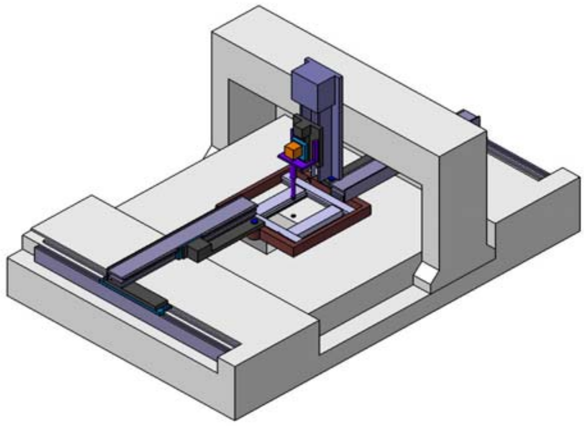


Fig. 10. Conceptual design of newly developed depaneling system

VII. Conclusion

A new type of planar three-degree-of-freedom parallel mechanism has been presented in this paper. This has been designed to obtain a large workspace, high speed, and compact structure. The kinematics problems, inverse and forward kinematics, were studied. All of them were expressed in closed form from its mechanical structure with decoupled-motion architecture. From the Jacobian matrix, the singularity problem was analyzed. To solve this singularity problem, the redundant actuation was addressed as a solution. From workspace analysis based on the analysis of the joint ranges, the reachable and dexterous workspace were obtained. The dexterous workspace of this mechanism is larger than conventional parallel mechanism. The depaneling system that has punching system and part handling system using parallel mechanism was proposed. We made a conceptual design for new system. And we are dedicated in making a prototype for the depaneling system to confirm our kinematic analysis and performance. After confirmation of kinematic performance, we are going to conduct dynamic analysis. Like depaneling system, several devices will be developed based on this mechanism. The kinematics, singularity and workspace analyses presented in this paper can be helpful in the design of such devices.

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