INTERNATIONAL JOURNAL OF PRECISION ENGINEERING AND MANUFACTURING Vol. 10, No. 4, pp. 00-00

DOI XXX-XXX-XXXX

Design and Analysis of Decoupled Parallel Mechanism with Redundant Actuator

Sun Ho Kim[1], Dongsu Jeon[1], Hyun Pyo Shin[1], Woosung In[1] and Jongwon Kim[1,#]
1 School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, South Korea, 151-742
Corresponding Author / E-mail: jongkim@snu.ac.kr, TEL: +82-2-880-7138, FAX: +82-2-875-4848

KEYWORDS: Decoupled motion architecture, Motion planning algorithm, Motion simulator, Parallel kinematic mechanism, Redundant actuation

This paper presents a redundantly actuated six degrees-of-freedom parallel kinematic mechanism with a partially decoupled architecture in its rotational motion. This mechanism is developed to eliminate kinematic complexity of original Eclipse-II known as a redundant parallel mechanism. Since the original Eclipse-II mechanism use kinematic redundancy of parallel mechanism to achieve an advantage in enlarging the workspace of the system, it needs a motion planning algorithm to choose the specific control inputs to determine the desired motion trajectory. This motion planning algorithm causes difficulty in achieving real-time control performance due to its structural complexity. However the redundant parallel mechanism presented in this paper is a redundant parallel mechanism with partially decoupled architecture in its rotational motion. Therefore modified Eclipse-II redundant parallel mechanism can realize effective real-time control performances and continuous 360-degree rotational motion in any direction of the moving platform with six degrees of freedom.

Manuscript received: February 4, 2008 / Accepted: June 22, 2009

NOMENCLATURE

A_i = Prismatic joints along the horizontal circular guide
B_i = Spherical joints on the moving platform
C_i = Prismatic joints on the vertical columns (i = 1,2,3)
$\{F\}$ = Fixed global reference frame
$\{M\}$ = Moving frame
α, β, γ = *ZYX* Euler angles of rotation about the three joint axes of rider.
$\alpha', \beta', \gamma', \delta'$ = *ZYXZ'* Euler angles of rotation about the four joint axes of chair on the moving platform of the Eclipse-II one-man ride machine.
$\alpha'', \beta'', \delta''$ = *ZYX* Euler angles of rotation about the three joint axes of chair on the moving platform of the modified Eclipse-II one-man ride machine.
$\boldsymbol{R}_{command}$, $\boldsymbol{R}_{control}$, $\boldsymbol{R}_{modified\_control}$ = Rotation matrices

1. Introduction

Parallel mechanisms consist of several serial chains connecting a base to a moving platform. Due to their structures, parallel mechanisms are capable of very fast and accurate motions, they possess higher average stiffness characteristics throughout their workspaces, and they can carry heavier payloads than serial mechanisms in general.[7] However, these advantages come at the expense of a reduced workspace, singular points in the configurations, difficult mechanical designs to achieve, and more complex kinematics and control algorithms than their serial counterparts.[5]

A singular configuration is the main drawback of a parallel mechanism. This is a configuration in which the degrees of freedom of the parallel mechanism changes instantaneously. Since organizing the links in a singular configuration can disable effective control of the device which can even break the mechanism and limit movements, these singular points must be eliminated from the workspace of the mechanism.[8-10] One method to eliminate the singular configurations is by using redundancy in the actuation and this can be classified into three types:[4] adding more actuators on the passive joints, adding additional branches (serial chains) with actuated joints, and adopting both these types of additional actuators.[6,9,11]

The Eclipse-II mechanism is a parallel mechanism capable of a 360-degree rotational motion of the moving platform as well as translational motion, which is designed for the motion base of the motion simulator. This mechanism was originally designed with six degrees of freedom, however, two additional degrees of freedom of kinematic redundancy have been added to eliminate the end-effector's singular configurations and to improve the capability of the rotational motion. In addition, one additional actuator is added to a passive joint to eliminate the actuator's singular configuration and finally the Eclipse-II mechanism is modified to possess eight degrees of freedom and nine axes parallel mechanism.[1]

The Eclipse-II one-man ride machine is developed primarily for use in motion simulator applications such as flight simulators and roller coaster simulators for recreational use. It is developed based on the results of theoretical analysis for the Eclipse-II mechanism with actuation redundancy and experimental verification using a working sample machine.[2] Despite it being able to realize any motions with continuous 360-degree rotational motion in any direction with six degrees of freedom, it has problems in achieving real-time motion planning and control performance due to its structural complexity.

The modified Eclipse-II one-man ride machine is presented here, in order to overcome these problems of achieving real-time algorithms. The modification is performed to enable three rotational axes of the moving platform to be controlled by independent actuators, on alternates axes using the redundant actuators. In this way, the Eclipse-II one-man ride machine has a partially decoupled architecture in its rotational motion. As a result, the modified Eclipse-II one-man ride machine can be controlled in real-time by its control algorithm.

In this paper, a novel six degrees-of-freedom parallel mechanism is presented; this mechanism is referred to as the Eclipse-II and it has nine actuators with three being redundant. The paper is organized as follows. In Section 2, the kinematic structure of the Eclipse-II with its characteristics of redundant actuation is presented. Section 3 describes the design and development issues of the Eclipse-II one-man ride machine, and section 4 presents the modified structure followed finally by some concluding remarks in Section 5.

2. 6-DOF Redundant Parallel Mechanism: Eclipse-II

2.1 Basic Kinematic Structure

The architecture of the original Eclipse-II mechanism is shown in Fig. 1. The Eclipse-II consists of three *PPRS* serial sub-chains that move independently on a fixed circular guide. Here, *P*, *R* and *S* denote prismatic, revolute and spherical joints, respectively. The Eclipse-II has six degrees of freedom. The six actuated joints are three prismatic joints (A_i) along the horizontal circular guide, two prismatic joints (C_2, C_3) on the vertical columns and another one prismatic joint (C_1) on the vertical circular column. All six actuated joints can be found in Fig. 1 and are indicated by arrows. One end of each connecting link B_iC_i is attached to the circular and vertical columns through the revolute joint. The other end of these links is mounted to the moving platform via a spherical joint (B_i).

Mounting the one circular column and the two linear columns on the circular guide gives the Eclipse-II a large orientation. The moving platform can rotate 360 degrees continuously about the y-axis in the moving frame $\{M\}$ and the Z-axis in the fixed frame $\{F\}$. Fig. 2 shows top view and side views of original Eclipse-II mechanism structure. More specific kinematic analysis showing its kinematic structure discussed in a previous paper.[1]

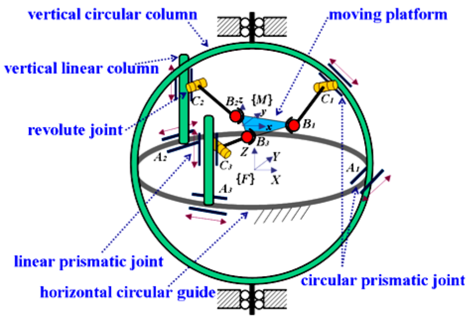
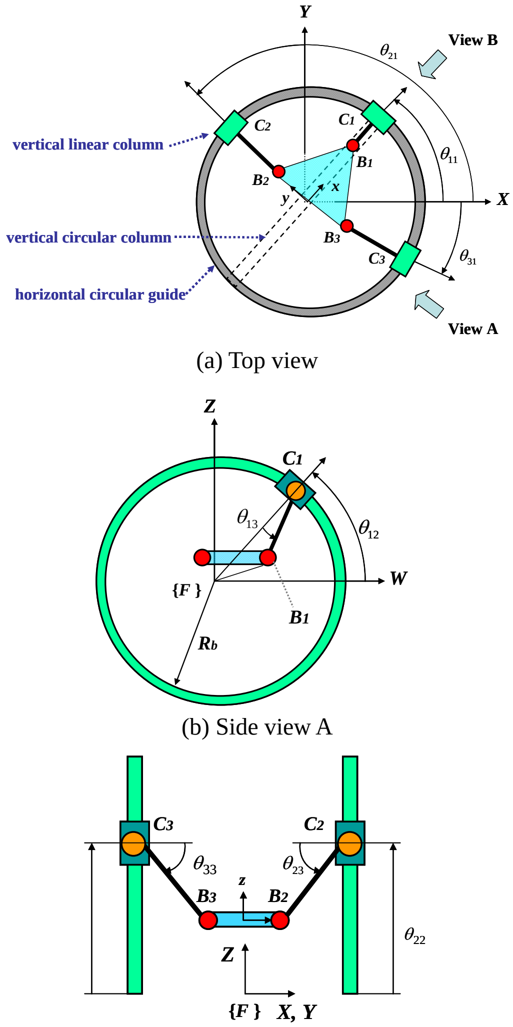


Fig. 1 Architecture of the Eclipse-II mechanism



(a) Top view

(b) Side view A

(c) Side view B

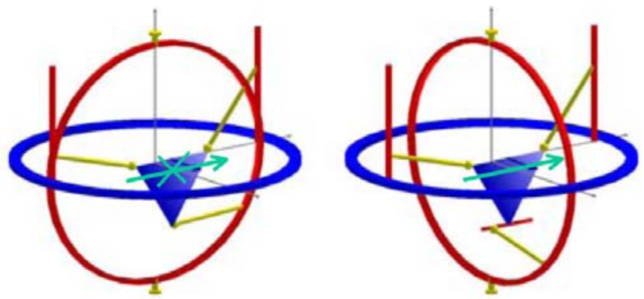
Fig. 2 Coordinate and joint convention

2.2 Singularity Analysis and Redundant Eclipse-II mechanism

In general, a parallel mechanism has two types of singularity:

end-effector singularity and actuator singularity.[3] In the Eclipse-II mechanism, these two types of singularity coexist in the workspace.[1]

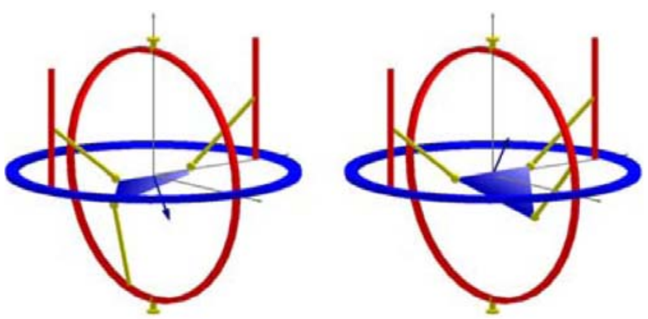
In the end-effector singularity of the Eclipse-II mechanism, as shown in Fig. 3(a), the moving platform cannot translate along the *y*-axis in the moving frame. This occurs in positions where one of the spherical joints is located on the *z*-axis of the fixed frame with the moving platform is rotated about the *y*-axis of the moving frame at 90° and 270°. The additional branches with actuated joints are added onto the moving platform to eliminate these end-effector singularities. This changes the position of the spherical joint that is connected to the circular column along the linear guide. As a result, the moving platform can move along the *y*-axis in the end-effector's singular configuration as shown in Fig. 3(b).



(a) 6-DOF Eclipse-II (b) (6+1)-DOF Eclipse-II

Fig. 3 *y*-direction motion of the Eclipse-II in the end-effector singular configuration

The actuator singularity of the Eclipse-II mechanism occurs in two positions; one is in the position where the moving platform has the rotation angle about the *y*-axis of the moving frame of 25° with the rotation angle about the *z*-axis of the moving frame of 0°, and the other is in the position where the moving platform has the rotation angle about the *y*-axis of the moving frame is 225° with the rotation about the *z*-axis of the moving frame is 180°. These actuator singularity configurations means that the moving platform cannot sustain its static equilibrium position in the presence of external forces,[12] and so move along an undesired direction as shown in Fig. 4. The extra actuators are added on the passive joints to eliminate the actuator singularities. In the Eclipse-II mechanism, one actuator is added on the passive revolute joint on one of the vertical columns.

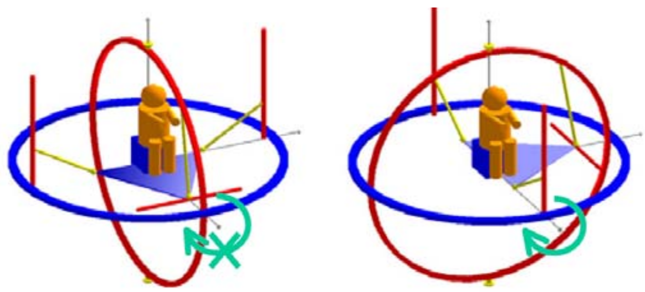


(a) angle about y-axis: 225°
angle about z-axis: 180°

(b) angle about y-axis: 25°
angle about z-axis: 0°

Fig. 4 Examples of the actuator singular configurations

In addition, one redundant actuator is added on the moving platform to improve the capability of the rotational motion. Since the Eclipse-II mechanism was originally designed for the motion base of the motion simulator, increasing the rotational motion capability is a crucial issue. The Eclipse-II mechanism is capable of a continuous 360-degree rotational motion of the moving platform about the *y*- and *z*-axes in the moving frame. In contrast, it has a limitation on its rotational motion about *x*-axis in the moving frame. The limitation of the rotational motion is improved by adding one redundant actuator to the bottom of the moving platform as shown in Fig. 5.



(a) (6+1)-DOF Eclipse-II (b) (6+2)-DOF Eclipse-II

Fig. 5 *x*-axis rotation of the Eclipse-II

As a result, the Eclipse-II mechanism becomes an eight degrees of freedom with two additional degrees of freedom of kinematic redundancy and nine-axis redundant parallel mechanism as shown in Fig 6. The theoretical analysis of the redundant Eclipse-II mechanism has been performed and verified by the working sample machine and specific discussions mentioned in a previous paper.[2]

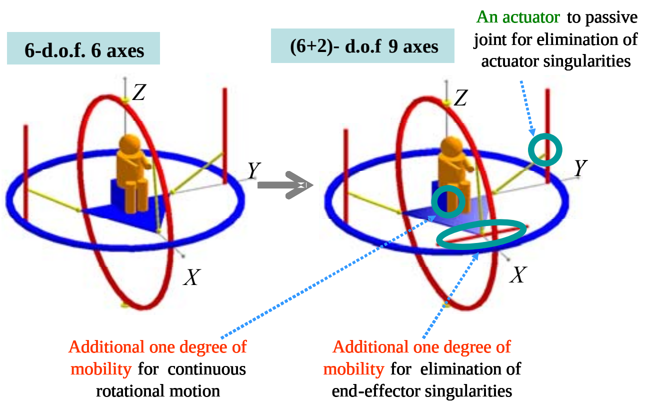


Fig. 6 (6+2)-DOF, 9-axis Eclipse-II mechanism

3. Eclipse-II One-man Ride Machine Development

The assembly drawings and photographs of the Eclipse-II one-man ride machine are shown in Fig. 7 and Fig. 8 and the device has been designed and manufactured based on experimental data of a working sample machine. The final system developed can be applied to emulate a motion simulator such as a flight simulator for military purpose, or a roller coaster simulator for recreational use. It is located in Seoul National University in Korea.

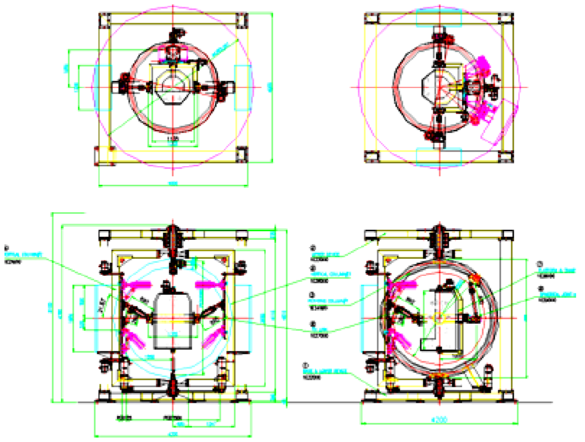


Fig. 7 Assembly drawing of the Eclipse-II one-man ride machine

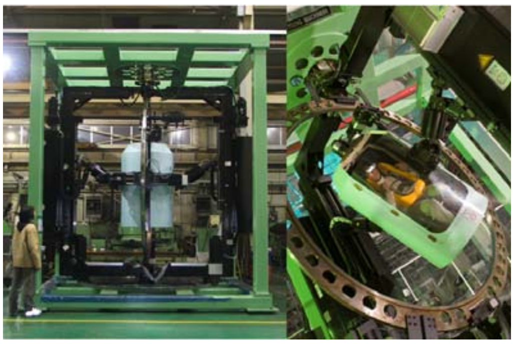


Fig. 8 Photographs of the Eclipse-II one-man ride machine

Table 1 Specifications of the Eclipse-II one-man ride machine

Overall size: 4,200 (L) × 4,000 (W) × 5,100 (H) mm
Platform size: 1,125 × 940 mm
Kinematic workspace: Ø236.6 × 491.3 mm
Max. linear speed: 36 m/min
Max. linear acceleration: 0.5g
Max. angular speed: 120 deg/sec
Max. angular acceleration: 500 deg/sec^2
Number of axes: 9
Actuators: AC servo motors

The main specifications of the Eclipse-II one-man ride machine are shown in Table 1. It can simulate any six degrees-of-freedom motion with a 360-degree rotational motion in any direction according to the prepared path through a washout filter and motion planning algorithm. Since it has two additional degrees of freedom kinematically as mentioned in Section 2, the motion planning and control algorithm is crucial in finding the optimal eight control inputs from the six degrees-of-freedom rider's command inputs. The basic motion planning algorithm for rotational motion is executed using rotational matrices of *z-y-x* and *z-y-x-z′* Euler angles as follows:

$$\boldsymbol{R}_{\text{control}} = \boldsymbol{R}_{\text{command}}$$

$$\boldsymbol{R}_{\text{command}} = \boldsymbol{R}_z(\alpha)\boldsymbol{R}_y(\beta)\boldsymbol{R}_x(\gamma) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

$$\boldsymbol{R}_{\text{control}} = \boldsymbol{R}_z(\alpha')\boldsymbol{R}_y(\beta')\boldsymbol{R}_x(\gamma')\boldsymbol{R}_z(\delta') = \begin{bmatrix} \cos\alpha' & -\sin\alpha' & 0 \\ \sin\alpha' & \cos\alpha' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta' & 0 & \sin\beta' \\ 0 & 1 & 0 \\ -\sin\beta' & 0 & \cos\beta' \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma' & -\sin\gamma' \\ 0 & \sin\gamma' & \cos\gamma' \end{bmatrix} \begin{bmatrix} \cos\delta' & -\sin\delta' & 0 \\ \sin\delta' & \cos\delta' & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where α, β and γ are the rotation angles about the *z, y* and *x* axes of the rider's frame, α', β', γ' and δ' are the rotation angles about the *z, y, x* and *z′* axes of the moving frame of the Eclipse-II one-man ride machine. The *z′* axis is for the additional rotational axis of the chair on the moving platform as Fig. 9 shows.

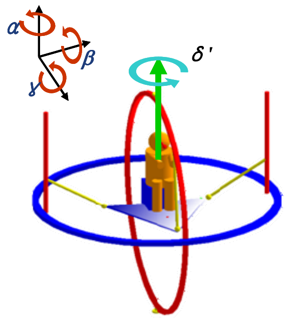


Fig. 9 Chair rotation of Eclipse-II mechanism

The input rotation matrix to be denoted as $\boldsymbol{R}_{\text{command}} \in \Re^{3\times3}$ in equation (1) is obtained from the rider's input. The output rotation matrix, $\boldsymbol{R}_{\text{control}}$ has the same dimension as the input matrix. Each component of the output matrix is expressed as a function of the output angles, α', β', γ' and δ', since there is additional chair rotation δ' according to *z′* axis. There are four unknown output variables, but only three of the nine components of the rotation matrix are independent. This causes the output angles to be non-unique and not to be defined. Essentially, there exists an infinite possible control inputs set because the redundant axis to be added along the *z*-axis in the moving frame is used to improve the limitation of the angles about the *x*-axis of the chair. It takes significant time to determine the optimal control inputs from the infinite candidates, because of iteration process considering the limited workspace and actuator capabilities as Fig. 10 shows. This figure describes the motion planning algorithm that finds optimal pose corresponding to the i^{th} input data, $\{X_i, Y_i, Z_i, \alpha_i, \beta_i, \gamma_i\}$ for a given path. In the iteration process of this algorithm, the control orientation value at each iteration step, {A, B, C} is calculated according to the value of δ which is increased gradually within the reachable angle of chair rotation. These value sets of $\{A, B, C, \delta\}$ can be expressed in the form of $\{\alpha'_{ij}, \beta'_{ij}, \gamma'_{ij}, \delta'_{ji}\}$. Among them, the optimal control orientation value which minimizes the joint torque can be selected.

The iteration process in motion planning algorithm causes difficulty in real-time motion planning and control for the Eclipse-II one-man ride machine which means it is only useful for the motion simulator that is operated by predefined paths, such as for the roller coaster simulations for recreational use, and real-time control with limited motions such as the flight simulation of an aircraft of civil aviation, tank and car simulator. Therefore the modification of the Eclipse-II one-man ride machine is to be considered in order to realize the real-time control.

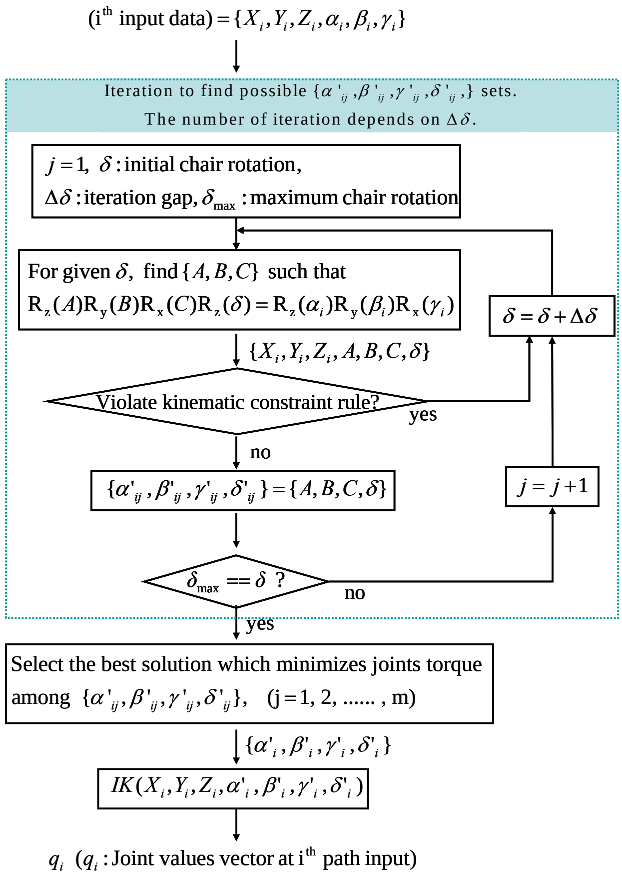


Fig. 10 Motion planning process for Eclipse-II one-man ride machine

4. Modification of Eclipse-II One-man Ride Machine for Flight Simulator

4.1 Modification for real-time control

A flight simulator needs a real-time control algorithm according to the inputs from pilot's motion through the control stick and pedals. In particular, a fighter simulator for military purpose needs real-time response for the extreme input motions such as turning movements with high acceleration.

Since the Eclipse-II one-man ride machine has difficulties in achieving real-time control performance, as mentioned in Section 3, a modified kinematic structure is proposed. The modification is performed to enable control of the three rotational axes of the moving platform by independent actuators each other using redundant actuators. This is known as a partially decoupled architecture in the rotational motion and can be achieved by using a redundant actuator attached on the chair of the moving platform and being charged controlling the rotational motion about the *x*-axis in the moving frame.

In the case of the original Eclipse-II one-man ride machine, the redundant actuator, which is used to rotate the chair on the moving platform, is used for the yaw motion, which is the rotational motion, about the *z*-axis in the moving frame $\{M\}$. However, in the case of the modified Eclipse-II one-man ride machine, the redundant actuator is charged for the roll motion that is the rotational motion about the *x*-axis in the moving frame as shown in Fig. 11. The modification substitutes a redundant axis for the previous rotational axis about the *x*-axis and enables roll, pitch and yaw motions to be controlled by separated independent actuators. Since the roll and pitch motions are controlled by only one actuator but the yaw motion is controlled by three actuators, the rotational motion is partially decoupled as equation (2), where α'', β'' and δ'' are the rotation angles about the *z, y* and *x* axes of a moving frame of the modified Eclipse-II one-man ride machine. Also the input command rotation and output control rotation can be directly matched as equation (3). As a result of the modification, the motion planning algorithm for the rotational motion is not required any more, as shown in Fig. 12, and Fig. 13 shows the comparison of structure between the Eclipse-II mechanism and the modified mechanism.

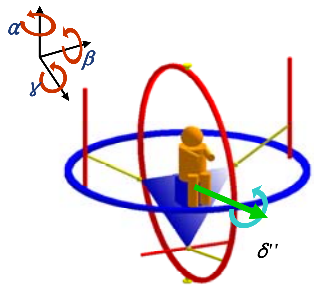


Fig. 11 Chair rotation of modified Eclipse-II mechanism

$$\boldsymbol{R}_{\text{modified\_control}} = \boldsymbol{R}_{\text{command}}$$
$$\boldsymbol{R}_{\text{command}} = \boldsymbol{R}_z(\alpha)\boldsymbol{R}_y(\beta)\boldsymbol{R}_x(\gamma)$$
$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$
$$\boldsymbol{R}_{\text{modfied\_control}} = \boldsymbol{R}_z(\alpha'')\boldsymbol{R}_y(\beta'')\boldsymbol{R}_x(\delta'')$$
$$= \begin{bmatrix} \cos\alpha'' & -\sin\alpha & 0 \\ \sin\alpha'' & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta'' & 0 & \sin\beta'' \\ 0 & 1 & 0 \\ -\sin\beta'' & 0 & \cos\beta'' \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta'' & -\sin\delta'' \\ 0 & \sin\delta'' & \cos\delta'' \end{bmatrix} \quad (2)$$

$$\alpha = \alpha'', \quad \beta = \beta'', \quad \gamma = \delta'' \quad (3)$$

This enables the modified Eclipse-II one-man ride machine to be operated by real-time motion planning and control. The modified real-time motion planning process is described in Fig. 11. In fact, the other additional actuator used to eliminate the workspace singularity causes some motion planning problems. Moreover, the motion considering both the translation and rotation is coupled and makes control of the machine difficult. However the motion planning algorithm is used only for the positions of the workspace singularities not for the whole positions. In addition, the control of the modified machine is less complex than the original one since the rotational motions that are mainly used in the motion simulator applications are controlled by partially decoupled actuators. Due to these reasons, the modified Eclipse-II one-man ride machine with the partially decoupled motion architecture in the rotational motion can be controlled in real-time. The modified Eclipse-II one-man ride machine has been designed and manufactured as shown in Fig. 14.

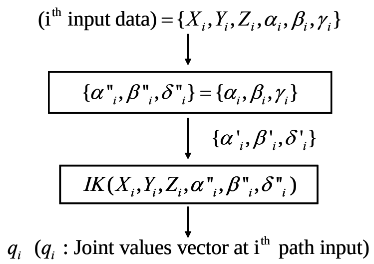
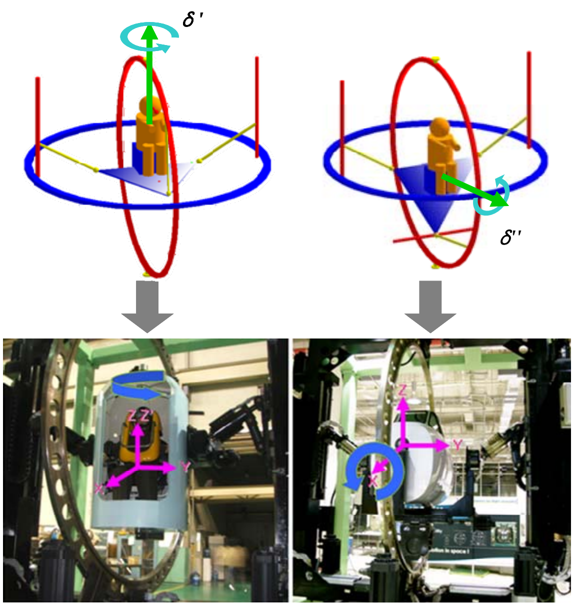


Fig. 12 Motion planning process for modified Eclipse-II one-man ride machine



(a) The Eclipse-II (b) The modified Eclipse-II

Fig. 13 Comparison between Eclipse-II mechanism and modified Eclipse-II mechanism

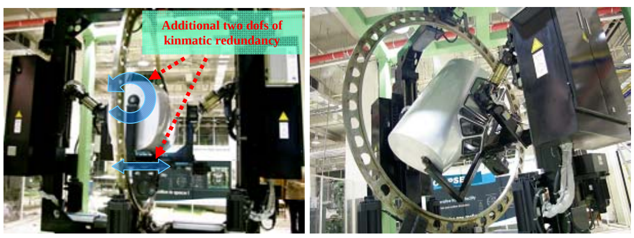


Fig. 14 Photograph of the modified Eclipse-II one-man ride machine

4.2 Kinematics of the modified Eclipse-II one-man ride machine

Since the kinematic structure of the Eclipse-II one-man ride machine has been modified, its kinematic analysis for the modified mechanism including the singularity analysis and workspace analysis, is required. The changed kinematic parameters of modified Eclipse-II one-man ride machine are shown in Table. 2. For the singularity analysis, there is no difference between the singularity positions and the original mechanism. Therefore, the same redundant actuation technique is performed to avoid singularity configurations. In the case of the machine's workspace, this is changed because the chair size on the moving platform has been changed and the size of the frames and links has also been changed corresponding to the chair size. The size has been changed from Ø 940 × 1,125 (mm) to Ø 1,200 × 1,600 (mm). According to the modified chair size, the result of the workspace analysis is shown in Fig. 15. As a result, the workspace is changed from Ø 236.6 × 491.3 (mm) to Ø 300 × 280 (mm). This workspace is a dexterous one that is defined as the positions where the moving platform can reach for all the required orientations.

Table 2 Modified kinematic parameters

kinematic parameters	Eclipse-II one-man ride machine	Modified Eclipse-II one-man ride machine
Cabin (mm)	Ø1,000 × 1,600	Ø1,200 × 1,600
Distance between center of cabin B_2, B_3 joints (mm)	662.5	787.5
B_1C_1 link length (mm)	930	798.9
Set up angle of B_2, B_3 joints (normal vector)	B_2: [0, 1, 0]	B_2: [0, cos14°, sin14°]
	B_3: [0,-1, 0]	B_3: [0, -cos14°, sin14°]

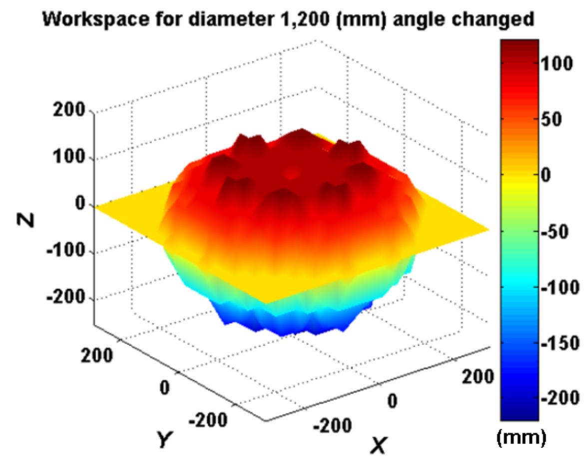


Fig. 15 Workspace of the modified Eclipse-II one-man ride machine

5. Conclusions

This paper presents a redundant 6 degrees of freedom parallel mechanism with partially decoupled architecture modified from the Eclipse-II. The original Eclipse-II mechanism has two kinematic redundancies to eliminate the singularities and increase the capability of the rotational motion, but it has difficulties in real-time motion planning and achieving effective control. To perform real-time motion planning, the modified Eclipse-II mechanism which is partially decoupled in rotational motions is proposed. The modification is performed to enable three rotational axes of the moving platform to be controlled by independent actuators, on alternates axes using the redundant actuators. The modified Eclipse-

II can perform continuous 360-degree rotational motion in any direction of the moving platform is possible in addition to translational motions. According to the modified mechanism, the modified Eclipse-II one-man ride machine is developed. The modified Eclipse-II one-man ride machine can be controlled in real time to reproduce any motions that cannot be realized in the previous existing motion simulator.

ACKNOWLEDGEMENT

This research was supported by the second stage of Brain Korea 21 program for Seoul National University.

REFERENCES

1. Kim, J., Hwang, J.-C., Kim, J.-S., Iurascu, C. C., Park, F. C. and Cho, Y. M., “Eclipse-II: A New Parallel Mechanism Enabling Continuous 360-degree Spinning Plus Three-axis Translational Motions,” IEEE Transactions on Robotics and Automation, Vol. 18, No. 3, pp. 367-373, 2002.

2. Kim, J., Cho, Y. M., Park, F. C. and Lee, J. M., “Design of parallel mechanism platform for simulating general six degrees-of-freedom motions including continuous 360-degree spinning,” Annals of the CIRP, Vol. 52, No. 1, pp. 347-350, 2003.

3. Park, F. C. and Kim, J. W., “Singularity Analysis of Closed Kinematic Chains,” ASME Journal of Mechanical Design, Vol. 121, No. 1, pp. 32-38, 1999.

4. Nokleby, S. B., Fisher, R., Podhorodeski, R. P. and Firmani, F., “Force capabilities of redundantly-actuated parallel manipulators,” Mechanism and Machine Theory, Vol. 40, No. 5, pp. 578-599, 2005.

5. Chung, Y. -H., Lee, J. -W., Sung, Y. -G. and Joo, H. -H, “Dynamics and Control of 2 DOF 5-bar Parallel Manipulator with Closed Chain,” IJPEM, Vol. 2, No. 1, pp. 5-10, 2001.

6. Jeong, J. I., Kang, D., Cho, Y. M. and Kim, J., “Kinematic calibration for redundantly actuated parallel mechanisms,” ASME J. of Mechanical Design, Vol. 126, No. 2, pp. 307-318, 2004.

7. Oiwa, T., “An Ultraprecise Machining System with a Hexapod Device to Measure Six-Degree-Of-Freedom Relative Motions Between the Tool and Workpiece,” IJPEM, Vol. 8, No. 2, pp. 3-8, 2007.

8. Kang, C-G., “Online Trajectory Planning for a PUMA Robot,” IJPEM, Vol. 8, No. 4, pp. 16-21, 2007.

9. Kim, J, Park, F. C., Ryu, S., Kim, J., Hwang, J. C., Park, C. and Iurascu, C., “Design and Analysis of a Redundantly Actuated Parallel Mechanism for Rapid Machining,” IEEE Transactions on Robotics and Automation, Vol. 17, No. 4, pp. 423-434, 2001.

10. Merlet, J. -P., “Singular Configurations of Parallel Manipulators and Grassmann Geometry,” The Int. J. of Robotics Research, Vol. 8, No. 5, pp. 45-56, 1989.

11. Park, F. C. and Kim, J. W., “Singularity Analysis of Closed Kinematic Chains,” ASME J. of Mechanical Design, Vol. 121, No. 1, pp. 32-38, 1999.

12. Angeles, J., “The Design of Isotropic Manipulator Architectures in the Presence of Redundancies,” The Int. J. Robotics Research, Vol. 11, No. 3, pp. 196-201, 1992.