39th International Symposium on Robotics 2008
Seoul, Korea / October 15~17, 2008

Gain-scheduled Robust Control for a Fine Actuation System of a 3-DOF Micro Parallel Positioning Platform

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Abstract

This paper presents the synthesis of gain-scheduled robust controller for the fine actuation system of parallel mechanism platform. The system modelling is conducted based on the physics-based modelling and the unknown parameters are estimated by experiments using sinusoidal inputs. The mixed-sensitivity H_∞ controller is synthesized based on the frequency response model. The weightings for synthesizing are designed to guarantee the tracking ability and disturbance rejection. The kinematics changes are compensated by gain-scheduling by synthesizing independent controllers in each poses. The resulting controllers are evaluated by extensive experiments.

1. Introduction

Recently, as the needs of micro products are increasing, more researches have done on the micro positioning platform. The developed micro positioning platforms are used in the field of semiconductor fabrication, micro laser machining, and so on. The micro products have advantages of space, material, energy, and cost.

Positioning platforms based on the parallel kinematic mechanisms are suggested due to the advantages of adopting micro positioning such as error accumulation and resolution [1]. The developed positioning platform is also suggested to realizing the micro laser machining without setup changing [2].

The control plays an important role of guaranteeing the micro-scale positioning. Several attempts have been made to enhance the positioning accuracy by adopting control theories. Kim et al. have implemented sliding mode controller with perturbation compensator to 2-degree of freedom (DOF) parallel machine [3]. Ryu et al. have suggested the idea of reversing the sign of integral gain when the direction of movement is reversed to reduce the tracking error [4].

This paper presents the controller synthesis and performance evaluation for fine actuators of 3-DOF micro parallel positioning platform. The positioning platform adopts the dual stage servo system to realize both long stroke and high accuracy. To realize fast and precise motions, a controller should be synthesized and verified. Especially, the controller for fine actuators is more important than that of coarse actuators because the transient and steady-state motions depend mostly on the motion of fine actuators.

We employed a gain-scheduled H_∞ controller to robustly ensure the desired accuracy of the fine control system. We derived a physics-based dynamic model for the fine system from a non-parametric system identification in order to match the experimental data well. The H_∞ controller was synthesized based on the dynamic model of home position. To guarantee the robustness of performance along to the pose change, the gain-scheduling method was applied based on the change of kinematic relation.

Section 2 describes the structure of the parallel mechanism platform. Section 3 presents the procedure of system modeling and the resulting frequency response model. Section 4 describes the procedure of synthesizing the gain-scheduled H_∞ controller. Conclusions are presented in Section 5.

2. Parallel mechanism platform

The parallel mechanism platform consists of movable platform, base platform, and three links that connect the two platforms. The left photo in Figure 1 describes the parallel mechanism. The resulting platform has 3- DOF motions of x-, y- translation and α-rotation. The platform is a meso-scale rectangular shape whose size is 20 × 23 mm. The stroke is 5 mm for both the x- and y-axis and 100 degrees for the α-axis. The platform is actuated by the three sets of two-stage linear actuators: a linear motor for rough positioning and a piezo actuator for fine positioning. The pose of end-effector is measured by combinations of three linear scales [5]. The platform is already assembled and the feature is shown in the right of Figure 1.

3. System modeling

The dynamic formulation is shown in Figure 2. Each piezo actuators are assumed to be the mass-spring-damper system. The masses of links and end-effector are relatively lighter than the mass of each piezo actuation part, and so the masses are not included in the dynamic formulation. The dynamic equations for each piezo actuation parts are as follows:

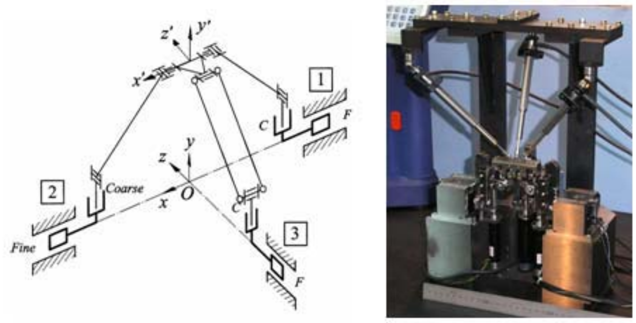
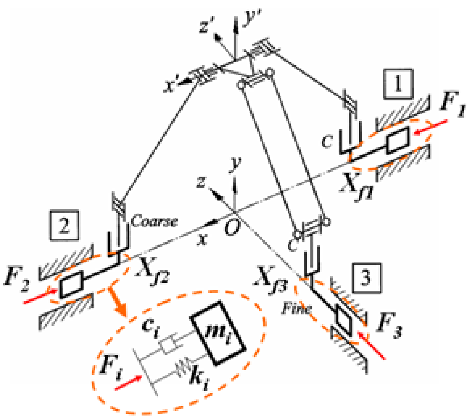


Figure 1 Parallel mechanism and assembled platform



m_1, m_2, m_3 = Masses of fine actuation part
c_1, c_2, c_3 = damping coefficients of fine actuation part
k_1, k_2, k_3 = stiffness coefficients of fine actuation part
F_1, F_2, F_3 = Generated force from fine actuators
V_1, V_2, V_3 = Input voltage to fine actuators

Figure 2 Relation of force and dynamic responses

$$F_i - k_i X_{fi} - c_i \dot{X}_{fi} = m_i \ddot{X}_{fi} \tag{1}$$

By combining these independent dynamic equations and the Jacobian matrix representing the geometric constraints, we can get the state-space dynamic model of the micro parallel positioning platform as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}, \tag{2}$$

where

$$x = [X_{f1}\ \dot{X}_{f1}\ X_{f2}\ \dot{X}_{f2}\ Z_{f3}\ \dot{Z}_{f3}]^{\mathrm{T}},\ u = [V_1\ V_2\ V_3]^{\mathrm{T}},\ y = [x\ y\ \alpha]^{\mathrm{T}}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{k_2}{m_2} & -\frac{c_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\frac{k_3}{m_3} & -\frac{c_3}{m_3} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_3} \end{bmatrix} \times 10^{-3}, \tag{3}$$

$$C = \begin{bmatrix} J_{f11} & 0 & J_{f12} & 0 & J_{f13} & 0 \\ J_{f21} & 0 & J_{f22} & 0 & J_{f23} & 0 \\ J_{f31} & 0 & J_{f32} & 0 & J_{f33} & 0 \end{bmatrix}.$$

We can calculate the Jacobian matrix and measure the mass of end-effector but we cannot calculate the stiffness and the damping coefficients. To apply this model to synthesize the controller, the proper stiffness and damping coefficient are required. The parameters were identified by experiments. To ignore the nonlinear property like backlash, we used the sinusoidal inputs of several frequencies instead of the white noise. Then we measured the magnitude change and phase delay of each sinusoidal input, and obtained the transfer relation which is indicated by a star shaped points of blue, red and magenta colors in Figure 3. Then, we adjusted the damping and stiffness coefficient and checked the bode plot of the transfer matrix to match the state-space model and experimental result. To simplify the dynamic model, we assume that the mass, damping and stiffness coefficient are the same value in each link, respectively. The estimated mass, damping and stiffness coefficient are 0.804 kg, 0.00816 N/μm·sec and 1.00 N/ μm, respectively. The transfer matrix of the micro parallel positioning platform is shown in (4), and the frequency response of the model is shown as the solid lines in Figure 3.

$$\begin{bmatrix} x \\ y \\ \alpha \end{bmatrix} = \begin{bmatrix} \frac{621.9}{s^2+406.1s+49750} & \frac{-621.9}{s^2+406.1s+49750} & 0 \\ \frac{510.4}{s^2+406.1s+49750} & \frac{510.4}{s^2+406.1s+49750} & \frac{161.7}{s^2+406.1s+49750} \\ \frac{-16.52}{s^2+406.1s+49750} & \frac{-16.52}{s^2+406.1s+49750} & \frac{50.05}{s^2+406.1s+49750} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{4}$$

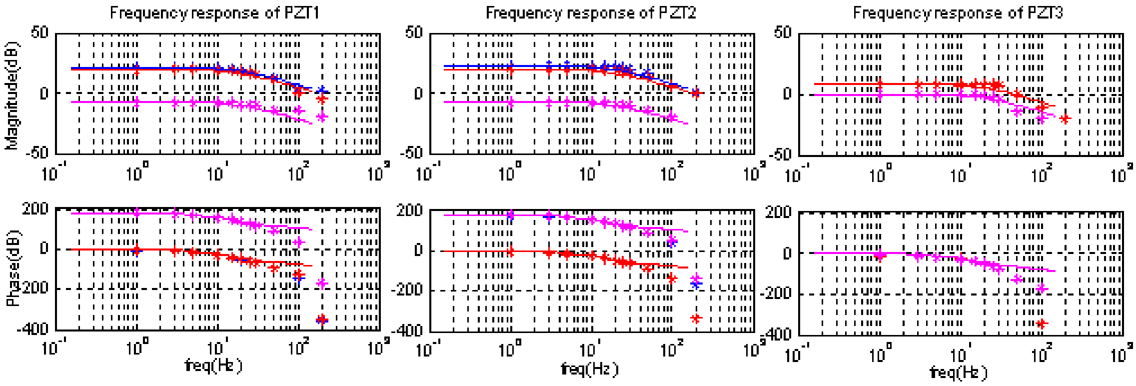


Figure 3 Frequency response of transfer functions and system identification results

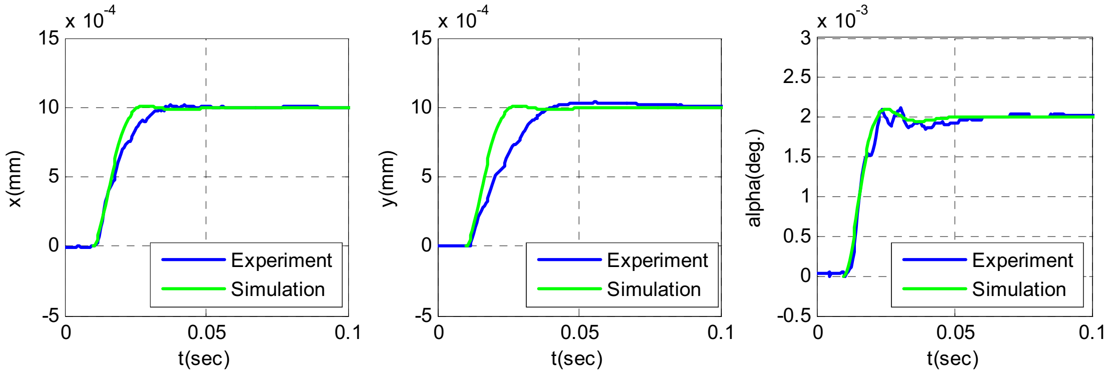


Figure 4 Step response of the H$_\infty$ controller

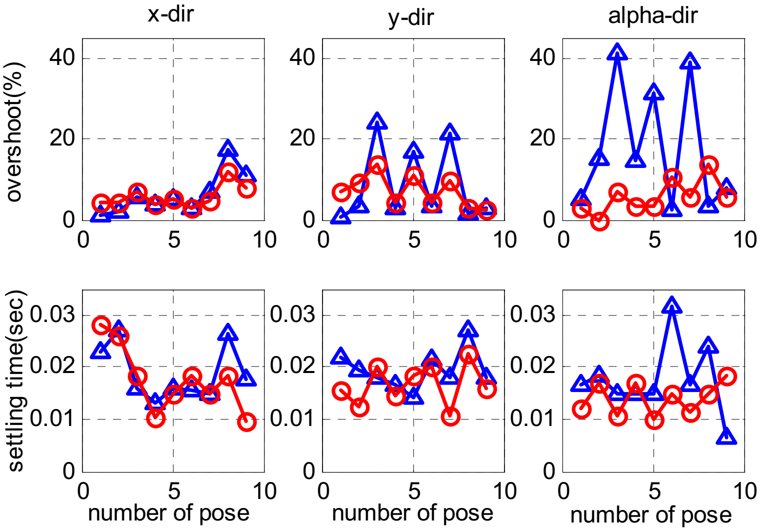


Figure 5 Experimental validation on effectiveness of the gain-scheduler

4. Synthesis of gain-scheduled H_∞ controller

4.1. Mixed sensitivity H_∞ controller

The mixed-sensitivity H_∞ controller is synthesized based on the model in (4). The weightings are designed to guarantee the -40 dB disturbance rejection at 1 Hz and the tracking frequency band of 35 Hz. The resulting weighting functions are as follows:

$$W_p = \begin{bmatrix} \frac{0.667s+157}{s+0.01} & 0 & 0 \\ 0 & \frac{0.667s+157}{s+0.01} & 0 \\ 0 & 0 & \frac{0.4s+157}{s+0.01} \end{bmatrix}, \tag{5}$$

$$W_t = \begin{bmatrix} \frac{s+285}{0.0316s+219} & 0 & 0 \\ 0 & \frac{s+285}{0.0316s+219} & 0 \\ 0 & 0 & \frac{s+285}{0.0316s+219} \end{bmatrix}, \ W_u = 10^{-3} I_{3\times3}$$

The challenge of the robust controller design is to choose a controller K such that the closed-loop transfer function, i.e., the complementary transfer function is robustly stable and the following condition (robust performance index) holds [6]

$$\left|W_p S\right|_\infty + \left|W_t T\right|_\infty < 1 \tag{6}$$

By using H_∞ synthesis algorithm, nine controllers are designed for the home position model. The step response is simulated and measured. For the fine actuators, we have used PX38SG (Piezojena). The step response of the controller from the simulation and experiment is shown respectively in Figure 4. 0.001 mm steps are used for translations and 0.002 degree step is used for rotation. From the experimental result, the controller shows 2.66%, 4.69% and 6.12% of overshoot and 0.016 sec, 0.022 sec and 0.010 sec of settling time in x-, y- and α-directional movement, respectively.

4.2. Gain scheduler

In the home position, we can get satisfactory performance of the controller by using the H_∞ controller. However, there is a problem in kinematics change along the pose change. Especially, the 100 degrees of α-directional rotational capability affects the kinematics change significantly, making the dynamic characteristics of closed-loop system to vary. So, we need to design a gain scheduler for the robust controller which is not sensitive to the pose change.

The procedure of designing gain-scheduler is very straight-forward. The frequency response models for each pose can be developed along to the procedure in Section 3. Based on the models, H_∞ controllers for each pose can be synthesized by using the same weightings in (5). The gain-scheduler function can be formulated by the DC gains of synthesized controllers considering the input/output relation of each element of controllers. The gain-scheduler function is as follows:

$$GS_F = \begin{bmatrix} -0.0081x^2 + 0.0069x + 0.9994 & -0.0165x^2 + 0.0134x + 0.9414 & 0 \\ 0.0039\alpha + 1.0786 & 0.0016a + 1.0673 & -0.0119\alpha + 0.8637 \\ 0.0578\alpha + 1.0685 \; or \; -0.0907\alpha - 1.1207 & 0.05\alpha + 1.0789 \; or \; -0.0920\alpha - 1.1003 & -0.0096\alpha + 0.9711 \end{bmatrix} \tag{7}$$

The reliability of gain-scheduler was evaluated by the experiments in the nine poses in the workspace based on the $L_9(3^4)$ orthogonal array. The overshoot and settling time of 0.001 mm and 0.002 degree step responses were selected as the result values to be analyzed. The results were analyzed by using the smaller-the-better S/N ratio. Figure 5 shows the result of analysis, where blue line denotes the result without gain-scheduler and red line denotes the result with gain-scheduler. In the case of overshoot, there is an improvement of 5.33 dB. In the case of settling time, there is an improvement of 1.31 dB.

5. Conclusion

The gain-scheduled H_∞ controller was successfully synthesized and implemented on the micro parallel positioning platform. The H_∞ controller was synthesized based on the frequency response model in the home position and gain-scheduler compensates the kinematics change in the workspace. The resulting controller was validated through the experiments of step response. The dual servo controller is being synthesized and implemented.

6. References

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