1 Introduction

Recently, redundantly actuated parallel mechanisms have been studied because of their advantages such as no actuator singularities [1], which allows more workspace and additional stiffness, which is generated by the internal preload assignment [2]. The stiffness is enhanced by assigning an internal torque distribution among the actuating joints and, therefore, is dependent on the number of additional actuators [3].

The stiffness and workspace of the mechanism are important design factors of a redundantly actuated parallel mechanism. For example, haptic devices usually require a software routine to control their stiffness and also a large workspace at the same time [4–6]. These two factors are also related to each other, and the stiffness is expected to increase by the kinematic parameter optimization. Xu and Li [7] used the particle swarm optimization method in the stiffness optimization of a 3-DOF parallel kinematic machine. Xu and Li [8] also investigated the stiffness of a 3-DOF translational parallel manipulator by screw theory and discussed the influences of the change in kinematic parameters on the stiffness of the manipulator. Lee et al. [9] used the composite design index to optimize the workspace of a five-bar redundantly actuated system. Thus, in the design process of a redundantly actuated parallel mechanism, we should consider how much active stiffness can be guaranteed by assigning an internal torque distribution. At the same time, the workspace of the mechanism should be maximized. However, the relationship between the workspace and stiffness is contrary to each other according to previous researches [10,11].

In this study, the stiffness and the workspace of a planar two degrees of freedom (2-DOF) parallel manipulator are optimized. The mechanism, that is, the manipulator, has three actuators although it needs only two actuators for control. Thus, the one additional actuator is used to control the stiffness of the mechanism [3].

The Taguchi method, whose procedure is depicted in Fig. 1, was introduced in order to conduct the optimization. This method was originally developed for quality engineering and is a tool for systematically optimizing a given criterion by experiments or simulations [12].

For our problem, a stiffness matrix, which is expressed as a Hessian matrix is used. The Hessian matrix has three dimensional elements obtained from the derivative of the Jacobian matrix and it exists in case of redundant actuation situation. To examine the relationship between kinematic parameters and stiffness, partial differentiation of the Hessian matrix needs to be performed with respect to the kinematic parameters for the given mechanism. Moreover, additional Jacobian operation is also required to obtain the matrix since the dimension of the Hessian matrix is already three. As a result, complexity should be increased in proportion to the number of kinematic parameters and dimension.

Instead of formulating a numerical procedure, here, we optimized the workspace and mean stiffness by using the Taguchi, which is easy to apply. Compared with the analytic approach, the Taguchi method requires only the output response from the input and feedback without increasing the dimension of the matrix. Especially, the numerical approach for optimization is not appropriate for early stages of design because of heavy numerical cost.

The Taguchi method for design of experiments was conducted by several previous researchers. Lee et al. [12] used the Taguchi method to size the actuators of a 6-DOF parallel mechanism. Lee and Kim [13] used the Taguchi method in proportional-integral-derivative (PID) controller gain tuning of a 6-DOF parallel mechanism.
mechanism. Rout and Mittal [14] used the analysis of variance (ANOVA) technique to find the parameter set that can minimize the mean positional error of a 2-DOF revolute-revolute (RR) planar manipulator. 

Herein, we used the Taguchi method to examine the relationship of the kinematic parameters and to obtain quasi-optimal solution of a 2-DOF parallel manipulator to ensure the maximum mean stiffness while considering workspace simultaneously.

The related research on the performance of parallel mechanisms in view of wrench capability analysis has been done. Garg et al. [15] studied the maximum applicable force and associated moment of 3-revolute-revolute-revolute-spherical (3-RRRS) redundantly actuated parallel mechanism in various positions. Nokleby et al. [16] showed the improvement of force capability in redundant actuation by using scaling factor method. They used maximum and minimum values of performance index.

The objective of this study mainly focuses on the examination of the change of the mean stiffness and the workspace with respect to the change in the kinematic parameters of the mechanism and focuses on extracting dominant kinematic parameters that influence stiffness and workspace of the given mechanism. We used mean value of stiffness by averaging stiffness in total workspace, which includes active stiffness and passive stiffness at given positions of the mechanism.

The response analysis of the Taguchi method was conducted to reduce the number of experimental variables, that is, eleven controllable factors to the four most influential factors. The L_{27} (3^{13}) orthogonal array was used to reduce the time spent in the first simulation process. After conducting the response analysis, the L_{0}

![Fig. 1 Optimization procedure in Taguchi method](image1.png)

![Fig. 2 Schematic diagram of the planar 2-DOF parallel manipulator](image2.png)

(3^4) orthogonal array was applied with the selected four controllable factors. Through the optimization procedures, we found several kinds of optimal kinematic parameter level groups. From the results, we derived the relationship between the various link lengths and the change in workspace and mean stiffness of the planar 2-DOF parallel manipulator.

### 2 Kinematic and Jacobian Analysis

In our research, we used a planar 2-DOF parallel manipulator [3,17] consisting of five links \( l_i \) and a tool plate. It also has eight revolute joints, namely, three ground joints \( B_i \) two intermediate joints \( S_{ij} \) and three tool plate joints \( P_i \). The mechanism has two kinematic degrees of freedom and three joints that are actuated. A schematic diagram of the planar 2-DOF parallel manipulator is depicted in Fig. 2.

Constraint and forward Jacobian are the relationships between the time derivatives of the input variables and output variables of the kinematic expressions of the constraint equation and forward equation, respectively. \( \mathbf{q}_{all} \) represents the total joint vector. \( \mathbf{q}_i \) is an angular position vector of independent joints. \( \mathbf{q}_i \) is an angular position vector of dependent joints. \( \mathbf{q}_i \) is an angular position vector of actuation joints. A total of eight joints exist in the planar 2-DOF parallel manipulator. However, the last three joints do not appear in the \( \mathbf{q}_{all} \) joint vector because they are used as a constraint condition in the kinematic analysis.

\[
\mathbf{q}_{all} = [q_1, q_2, q_3, q_4, q_5]^T
\]

where \( \mathbf{U} \) is a relocation matrix, which rearranges the order of the independent and the dependent joint vector. \( \mathbf{V} \) is a selection matrix, which chooses the actuating joint vector among the \( \mathbf{q}_{all} \) vectors. The matrix \( \mathbf{U} \) and \( \mathbf{V} \) are obtained from below relationships, Eqs. (4) and (5).
The forward Jacobian matrix $J_f$ of Eq. (10) can be used to obtain the relationship between the active joint torque $\tau_a$ and the exerted external forces $f$ in the stiffness analysis.

### 3 Stiffness Analysis

Generally, the torque required to operate the parallel mechanism $\tau_a$ can be determined uniquely in nonredundant actuation [6]. However, in the case of redundant actuation, the torques $\tau_a$ of the actuators are not determined uniquely. The relationship between the independent joint torque $\tau_u$ and the actuated joint torque $\tau_a$ can be derived by the virtual work theorem. The relationship is presented in Eq. (12).

$$M\ddot{q}_u + C\dot{q}_u + N = \tau_u$$

$$\Gamma^T \tau_a = \tau_u$$

However, the actuating joint torque $\tau_a$ is not uniquely given but infinite numbers of the solutions can be derived since $\Gamma^T$ does not have full rank.

$$\tau_a = (\Gamma^T)^+ \tau_u$$

(13)

where the actuating joint torque of a redundantly actuated parallel mechanism can be divided into two parts. The first term is the motion torque, which causes the motion of the mechanism. The second term is the internal preload torque, which does not induce any motion of the mechanism. Mathematically, the internal torque solution vector exists in the null space of the Jacobian. The internal preload torque can increase the active stiffness of a redundantly actuated parallel mechanism and is obtained from below relationships.

$$\Gamma^T \tau_a = \begin{bmatrix} 1 & 0 & \Phi_{11} & \tau_1 \\ 0 & 1 & \Phi_{12} & \tau_2 \\ 0 & 0 & \Phi_{13} & \tau_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(14)

$$\tau_1 = -\Phi_{11} \tau_3$$

(15)

The influence of internal preload torque is maximized when one actuator torque of three actuator torques reaches its limit torque output. Maximum torque output is applied in the next section of optimization procedure in the active stiffness term.

Stiffness can be defined as the ratio of the exerted force to an infinitesimal displacement, which is shown in Eq. (16).

$$K = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

The relationship between the actuating joint torque of redundantly actuated parallel mechanisms and the force exerted on the platform of the mechanisms can be obtained by the virtual work theorem as in Eq. (17).

$$\tau_a^T dq_u = f^T dx_i$$

$$\tau_a^T \Gamma dq_u = f^T J dq_u$$

$$f = (\Gamma J_f^T)^+ \tau_a = \Psi^T \tau_a$$

(17)

Hence, the stiffness of redundantly actuated parallel mechanisms can be represented by partial differentiation of Eq. (18). The relationship can be written as follows:
As an example of a stiffness analysis, Fig. 3 is illustrated. The outermost contour displays the workspace of the manipulator, and the dotted circle is the radius of gyration (scaled down by half for convenience) to examine the distribution of the workspace.

4 Optimization Procedure: Taguchi Method and Simulation Planning

The Taguchi method was originally developed for quality engineering in order to evaluate and improve robustness of products, tolerance specification, and quality management of a production process [18]. It does not draw upon complicated probability or statistical analysis. The methodology can be applied to our problem, which requires kinematic parameter optimization through simulation. With this method, an optimized solution can be obtained by design of experiments, which normally demands complicated mathematical expansion in the theoretical approach.

The Taguchi method divides the independent variables into controllable factors and noise factors. Controllable factors can be maintained to a desired value while noise factors may not be controlled. The Taguchi method can realize a robust design, which can maintain high performance as well as insensitivity to noise factors. In this section, we apply the Taguchi method in designing the experiments (or in this case the simulations) for maximizing the workspace and the mean stiffness of the suggested parallel mechanism. Under the framework of the Taguchi method [18], the basic steps for designing an experiment or simulation are as follows:

A. Identifying the objectives: In the first step of the Taguchi method, identifying a specific objective is important because it determines the objective function and influences the classification of variables into controllable factors and noise factors. The stiffness of the planar 2-DOF parallel manipulator of our study was related to the lengths of the links and the installed position of the actuators. Therefore, the objective is to examine the relationship among the workspace, the mean stiffness, and the kinematic parameter level.

B. Determining the objective function: The Taguchi method classifies the objective function into one of three types: nominal-the-best, smaller-the-better, and larger-the-better (see Table 1). The objective function in our case was the sum of the workspace and mean stiffness of the manipulator, and it was a larger-the-better problem.

C. Selecting the controllable factors and noise factors: The factors to be tested for their influence on the objective function are selected. This procedure determines which orthogonal array should be used according to the selected controllable and noise factors.

Table 1 Typical S/N ratios

<table>
<thead>
<tr>
<th>Objective</th>
<th>S/N ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal-the-best</td>
<td>S/N = -10 log((\frac{1}{N}\sum_{i=1}^{N} y_i^2))</td>
</tr>
<tr>
<td>Smaller-the-better</td>
<td>S/N = -10 log((\frac{1}{N}\sum_{i=1}^{N} 1/y_i^2))</td>
</tr>
<tr>
<td>Larger-the-better</td>
<td>S/N = -10 log((\frac{1}{N}\sum_{i=1}^{N} 1/y_i^2))</td>
</tr>
</tbody>
</table>

\(y_i\) denotes the results of the ith turn of a set of n simulated trials.

As an example of a stiffness analysis, Fig. 3 is illustrated. Fig. 3 shows the workspace and the stiffness distribution of the planar 2-DOF parallel manipulator with the original kinematic parameters. The triangle and the connected lines constitute the kinematic structure of the manipulator. The contours and the numbers listed along the contours indicate the magnitude of the stiffness. The outermost contour displays the workspace of the ma-
optimal level (not always) of the selected factors could be investigated simultaneously. Thus, we set the number of levels for the lengths of the links and positions of the actuators to be three. Levels of the controllable factors were varied so that they could have the same variation ratio. The level of a noise factor was only one because the output displacement, which is the inverse of the stiffness, was proportional to the force exerted on the platform. The physical values assigned in this paper are given in Table 2. We set the levels of the controllable factors based on the kinematic parameters of the original 2-DOF manipulator. The force exerted on the platform (2.56 N) was set to maximum to guarantee linearity of the spring torsion bar mounted on the actuator.

D. Selecting an orthogonal array: A full factorial simulation requires the testing of all combinations of the factor levels. However, orthogonal arrays can be used to produce smaller but less costly simulations because it offers influence of each variable from least number of test times. There are several orthogonal arrays according to the combination of the level and the number of variables. It should be noticed that not all combinations exist. Therefore, we should select the orthogonal array, which has the most similar combination among the combinations that satisfy minimum requirement (that is level, number of variables). In this study, we used an L27 (3^11) orthogonal array. From the orthogonal array, first 11 columns are used and last two columns are ignored [18]. Mainly, in a full factorial simulation, 3^11 = 177,147 simulations involving 11 factors at three levels are required. Applying the L27 (3^11) orthogonal array, only 27 simulations need to be carried out (for details about the L27 (3^11) orthogonal array, see Table 6 in the Appendix).

E. Simulations and analysis: Simulation and analysis are developed in the planning and design stages. The analysis stage of the simulation relates to the calculations that convert raw data into the representative signal-to-noise ratio (S/N ratio). As a measurement tool for determining robustness, the S/N ratio is an essential factor in the optimization of design parameters.

By representing the impact of noise factors on the process or product as the denominator, the S/N ratio can be adopted as the index of how well the system performs regardless of the noise effects. Analysis also includes the determination of the most important controllable factors, which can maximize the S/N ratio, and the selection of their optimal levels. Index to be optimized is defined as follows:

\[ \text{S/N ratio} = \frac{M}{(j=1) w_i d_{ij}} \]

subject to \( A_{\text{act}} > 0.4A_{\text{ori}} \) (23)

where \( w_1 \) and \( w_2 \) are weight factors for the workspace and the mean stiffness. \( A_{\text{ori}} \) is the workspace area with the original parameter set-up, whereas \( A_{\text{act}} \) is the workspace area with the parameter of each simulation set-up. The workspace area \( A = M/n^2 \times 100 \) is computed by counting points \( M \) in the total grid (\( n \) by \( n \) matrix), which satisfies inverse kinematic constraints. \( S_{\text{ori}} \) is the mean stiffness with the original parameter set-up, whereas the \( S_{\text{act}} \) is the mean stiffness with the parameter of each simulation set-up. The mean stiffness \( S = (360M/\sum_{i=1}^{M} d_{ij}) \) is a reciprocal of an average displacement. The average displacement is obtained by averaging displacements of all points of the workspace when the external force \( f \) is applied in all directions (360 deg). \( d_{ij} \) is displacement, where \( i \) and \( j \) indicate degree and computation point in the workspace, respectively. This index is a unique input value of the S/N ratio and it can be used directly as the S/N ratio. In the case of the workspace, there is a constraint to prevent the workspace from decreasing below the specified area.

5 Response Analysis

The manufactured planar 2-DOF parallel manipulator is shown in Fig. 4 with the original dimensions, which are listed in Table 3. The lengths of five links were equal, and the third actuator was mounted a little apart from the center of the other actuators in the horizontal direction.

The number of design variables that had more influence than the other design values was found by response analysis of the Taguchi method. The design variables of the planar 2-DOF parallel manipulator included the length of the links and the position of the actuating joints. With the result of 27 simulations, we averaged S/N ratios for each level of the controllable factors, which were calculated based on the orthogonal array. For example, Eq. (24) represents the computation of the average S/N ratio of controllable factor A.

### Table 2 Levels of controllable and noise factors of first stage simulation

<table>
<thead>
<tr>
<th>Controllable factor</th>
<th>Level 1 (mm)</th>
<th>Level 2 (mm)</th>
<th>Level 3 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1x1</td>
<td>224</td>
<td>280</td>
</tr>
<tr>
<td>B</td>
<td>1x2</td>
<td>224</td>
<td>280</td>
</tr>
<tr>
<td>C</td>
<td>1x3</td>
<td>224</td>
<td>280</td>
</tr>
<tr>
<td>D</td>
<td>2x1</td>
<td>224</td>
<td>280</td>
</tr>
<tr>
<td>E</td>
<td>2x2</td>
<td>224</td>
<td>280</td>
</tr>
<tr>
<td>F</td>
<td>B1x1</td>
<td>−240</td>
<td>−300</td>
</tr>
<tr>
<td>G</td>
<td>B1x2</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>B1x3</td>
<td>240</td>
<td>300</td>
</tr>
<tr>
<td>I</td>
<td>B2x1</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>J</td>
<td>B2x2</td>
<td>120</td>
<td>150</td>
</tr>
<tr>
<td>K</td>
<td>B2x3</td>
<td>378</td>
<td>420</td>
</tr>
</tbody>
</table>

### Table 3 Original dimensions of the planar parallel manipulator

<table>
<thead>
<tr>
<th>Legend</th>
<th>Meaning</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Dimension of a side of tool platform</td>
<td>215</td>
</tr>
<tr>
<td>l</td>
<td>Length of link</td>
<td>280</td>
</tr>
<tr>
<td>B1</td>
<td>Position of first actuating joint</td>
<td>(−300, 0)</td>
</tr>
<tr>
<td>B2</td>
<td>Position of second actuating joint</td>
<td>(300, 0)</td>
</tr>
<tr>
<td>B3</td>
<td>Position of third actuating joint</td>
<td>(150, 420)</td>
</tr>
</tbody>
</table>
and for the level of the link length. In fact, at the second simulation, the S/N ratio of the link started to increase at the third level, and it is needed to maintain the minimum workspace constraint by applying bigger link lengths of the three links.

The influence of the average S/N ratio of the first four controllable factors is obvious. As the lengths of \( l_{11}, l_{21}, l_{31}, \) and \( l_{12} \) become longer, the workspace itself increases, naturally. However, the mean value of stiffness decreases when the lengths of the linkages increases. With respect to this point, the rate of decrease of mean stiffness is greater than the rate of increase of the workspace. Therefore, the average S/N ratio becomes smaller, as the lengths of \( l_{11}, l_{21}, l_{31}, \) and \( l_{12} \) increases.

Based on the result of the first simulation, we conducted an additional simulation with the four controllable factors \( l_{11}, l_{21}, l_{31}, \) and \( l_{12} \). We applied the L9 orthogonal array in this simulation. For details of the L9 orthogonal array, see Table 7 in the Appendix.

The magnitude of the force exerted on the platform was the same as that of the first simulation. We applied smaller values for the levels of the three links \( l_{11}, l_{21}, l_{31} \) but applied a bigger value for the level of the link \( l_{12} \) than those of the first simulation. The bigger value has applied to the link \( l_{12} \) because the S/N ratio started to increase at the third level, and it is needed to maintain the minimum workspace constraint by applying bigger link length. In fact, at the second simulation, the S/N ratio of the link \( l_{12} \) increased as the level increases.

Figure 6 shows the result of the response analysis about the average S/N ratios with respect to the four controllable factors, which are the lengths of four links \( l_{11}, l_{21}, l_{31}, \) and \( l_{12}. \) Among the four links, the third link \( l_{31} \) was the most influential factor. The average S/N ratios for the first three controllable factors, the lengths of links \( l_{11}, l_{21}, l_{31}, \) seemed to be similar to each other but the fourth controllable factor, the length of link \( l_{12}, \) was not similar to the average S/N ratios of the other three controllable factors. Links \( l_{11}, l_{21}, l_{31} \) were connected directly on actuators but link \( l_{12} \) was connected to link \( l_{11} \) indirectly.

6 Simulation Results and Discussion

The optimization procedure yielded new kinematic parameter groups (see Table 4). The new parameter group gave better performance than the original kinematic parameter group did with respect to the workspace and mean stiffness of the planar 2-DOF parallel manipulator.

The results of the simulation in the second stage are suggested in Table 5. In the results, the champion (quasi-optimal) group and the eighth group improved the performance of the manipulator by over 12% and 80%, respectively. The champion group showed distinctively high mean stiffness, and the eighth group showed high mean stiffness without much loss of the workspace. The detailed performance of each kinematic parameter group is depicted in Table 5, and the workspace and the stiffness of each case are depicted in Fig. 7. Figure 7 shows the workspace and the

Fig. 5 Response graph: average S/N ratios for the eleven controllable factors

\[
\eta(\text{level } A_1) = \frac{\sum_{i=1}^{9} A_{i1}}{9}
\]

\[
\eta(\text{level } A_2) = \frac{\sum_{i=1}^{9} A_{i2}}{9}
\]

\[
\eta(\text{level } A_3) = \frac{\sum_{i=1}^{9} A_{i3}}{9}
\]

Fig. 6 Response graph: average S/N ratios for the four controllable factors

Table 4 Levels of controllable and noise factors of the second stage simulation

<table>
<thead>
<tr>
<th>Controllable factor</th>
<th>Level 1 (mm)</th>
<th>Level 2 (mm)</th>
<th>Level 3 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ( l_{11} )</td>
<td>150</td>
<td>187</td>
<td>224</td>
</tr>
<tr>
<td>B ( l_{31} )</td>
<td>150</td>
<td>187</td>
<td>224</td>
</tr>
<tr>
<td>C ( l_{31} )</td>
<td>150</td>
<td>187</td>
<td>224</td>
</tr>
<tr>
<td>D ( l_{12} )</td>
<td>336</td>
<td>420</td>
<td>504</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Noise factor</th>
<th>Level (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Force exerted on the platform</td>
<td>2.56</td>
</tr>
</tbody>
</table>
TABLE 5 Result of the second stage simulation

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Controllable factors</th>
<th>Output (I\text{output}/I\text{original})</th>
<th>Link length (mm)</th>
<th>Workspace (m²)</th>
<th>Mean stiffness (N/m)</th>
<th>Radius of gyration (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1</td>
<td>1.769</td>
<td>11, 21, 33, 12</td>
<td>0.049</td>
<td>174.709</td>
<td>0.548</td>
</tr>
<tr>
<td>2</td>
<td>1 2 2 2 2</td>
<td>1.656</td>
<td>150, 150, 150</td>
<td>0.062</td>
<td>154.268</td>
<td>0.557</td>
</tr>
<tr>
<td>3</td>
<td>1 3 3 3 3</td>
<td>1.419</td>
<td>150, 224, 224</td>
<td>0.057</td>
<td>129.651</td>
<td>0.571</td>
</tr>
<tr>
<td>4</td>
<td>2 1 2 2 3</td>
<td>1.635</td>
<td>187, 150, 187</td>
<td>0.054</td>
<td>167.478</td>
<td>0.558</td>
</tr>
<tr>
<td>5</td>
<td>2 2 3 1 1</td>
<td>1.246</td>
<td>187, 224, 224</td>
<td>0.074</td>
<td>106.266</td>
<td>0.556</td>
</tr>
<tr>
<td>6</td>
<td>2 3 1 2 2</td>
<td>1.671</td>
<td>187, 224, 150</td>
<td>0.072</td>
<td>158.413</td>
<td>0.554</td>
</tr>
<tr>
<td>7</td>
<td>3 1 3 2 2</td>
<td>1.353</td>
<td>187, 224, 150</td>
<td>0.079</td>
<td>117.588</td>
<td>0.557</td>
</tr>
<tr>
<td>8</td>
<td>3 2 1 3 1</td>
<td>1.805</td>
<td>224, 150, 224</td>
<td>0.074</td>
<td>175.782</td>
<td>0.558</td>
</tr>
<tr>
<td>9</td>
<td>3 3 2 1 3</td>
<td>1.273</td>
<td>224, 224, 150</td>
<td>0.080</td>
<td>119.014</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>1 1 1 1 3</td>
<td>2.273</td>
<td>150, 150, 150</td>
<td>0.047</td>
<td>246.705</td>
<td>0.555</td>
</tr>
</tbody>
</table>

*Denotes champion group obtained from the second response analysis.

The stiffness graph from the second stage simulation (w₁ w₂ = 1:1). The contours plotted inside the workspace represent the stiffness of the manipulator.

Mean stiffness is computed by averaging the stiffness of all points in the total workspace. As mentioned, the triangle and the connected lines in Fig. 7 constitute the corresponding kinematic structure. A circle, which is displayed as a dotted line, represents the radius of gyration. For convenience, the magnitude of the radius is scaled down by half. The radius of gyration is represented as below.

\[
R_{\text{gyration}} = \sqrt{\frac{I}{A}} \quad I = \int r^2 dA
\]

where \( I \) is the second moment of inertia along the axis located in the center of an area and \( A \) is the workspace area. With the radius of gyration, we can investigate the workspace distribution around the centroidal axis of the workspace whether the workspace is distributed broadly or not.

The first group is composed of link parameters, which have first levels, and naturally its workspace is the smallest among the first eight groups. The radius of gyration is also the smallest among all the parameter groups, which means that the workspace is concentrated at the center of the area. The second group has a more broadly distributed workspace because of the longer links \( l_{21}, l_{31}, \) and \( l_{12} \) than those of the first group. The third group has link parameters of extreme values (level one or three) and thus, it nearly leads to the separation of the workspace into two parts. This result can be explained by the long radius of gyration, which comes from the scattered distribution of the workspace.

The fourth group has the same level of link \( l_{12} \) with the third group and middle level of links \( l_{11} \) and \( l_{31} \). Because of the lengths of the links \( l_{11} \) and \( l_{31} \), the fourth group has higher mean stiffness than the third group. The fifth group has the lowest mean stiffness whose levels of links \( l_{31} \) and \( l_{12} \) are opposite of those of the eighth group, which has the highest mean stiffness. The sixth group has a similar shape of workspace and radius of gyration to the fifth group, except for the relatively higher mean stiffness. The last three groups have similar combinations of link parameters, workspace, and radius of gyration. However, there are distinctive differences in the levels of links \( l_{31} \) and \( l_{12} \). Only the eighth group has a lower level of link \( l_{31} \) and higher level of link \( l_{12} \), which produce higher mean stiffness than those of the seventh and the ninth group.

The champion group (quasi-group denoted by * in Table 5 and Fig. 7) was obtained by taking the value of the control parameters with the highest S/N ratio in Table 5. It has incomparably high mean stiffness, which compensates the decrease in the area of the workspace. Actually, the workspace of the champion group is small and its geometrical shape is not so usable. The champion group, however, suggests that it is possible to produce a kinematic
parameter group, which has a better performance by combining the value of the control parameters with the highest S/N ratio when the geometrical constraint given to the workspace.

The lengths of the linkages have stronger influence on the workspace and the mean stiffness than the other kinematic parameters such as the position of the actuators. By analyzing the results, the tendency about the relationship of the link length to workspace and mean stiffness of the planar 2-DOF parallel manipulator can be observed.

1. As the length of the link connected directly to each actuator becomes shorter, the more the mean stiffness is increased. Similarly, as the length of the link connected indirectly to each actuator becomes longer, the more the mean stiffness is increased. The result demonstrates the Lever law [11]. The force exerted on the center of the platform is transferred to the actuator through the link directly attached. The torque produced by the exerted force is decreased or increased in proportion to the length of the link. Therefore, a shorter link can produce higher stiffness.

2. The shape of a workspace is related to the usable workspace defined by Liu et al. [10]. We evaluated the radius of gyration of each workspace to measure the distribution of the workspace. A small radius of gyration means that the workspace is distributed near the center of the workspace. The result shows that the long link contributes to a long radius of gyration and small usable workspace. In the case of the third group, the radius of gyration is longest, and the workspace is nearly divided into two parts, which means the workspace is less usable. Therefore, we should reduce the radius of gyration to ensure enough usable workspace.

3. The amount of change in workspace and mean stiffness had relatively low sensitivity to the install positions of the three actuators based on the result of the first response analysis. Therefore, the actuators can be installed in near positions in the base frame to minimize the space needed to mount the manipulator.

4. By adjusting link lengths, it is possible to organize a manipulator satisfying a variety of workspaces and mean stiffness without replacing actuators or changing the mount position of the actuators. The optimal kinematic parameter group is profitable for designing a manipulator that requires exceedingly high stiffness and not so large workspace such as a high precision-miniature part manufacturing machine tool. Also the eighth group is profitable for designing a manipulator that requires high stiffness and large workspace simultaneously such as a rapid prototyping machine tool.

With the observations mentioned above, it is possible to design an effective planar parallel manipulator, which has high stiffness as well as a relatively large workspace area.

7 Conclusions

We presented a kinematics parameter optimization that maximizes the workspace of the planar 2-DOF parallel manipulator as well as the mean stiffness inside the workspace.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Controllable factors</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1</td>
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The Taguchi method was applied in this optimization to examine the relationship of each link and to separate relatively influential kinematic parameters. The Taguchi method was designed to simplify optimization procedures and to reduce the number of simulations at the cost of precision in finding the optimal solution. The optimization minimized the decrease of the workspace but maximized the increase of the mean stiffness of the manipulator. Kinematic analysis and stiffness analysis were employed in the optimization. Based on the results, we found a new kinematic parameter level group.

Using an orthogonal array, we reduced the number of simulations. Through the first response analysis, the number of experimental variables was reduced to separate the more influential controllable factors from the less influential ones in the optimization procedure. By additional simulations, the optimal kinematic parameter group was obtained. We also discussed the relationships among the link length, the workspace, and the mean stiffness. As a future work, the adjoint variable method can be applied in response analysis, which is desired when the dimensions of design variables are larger than the number of response functions such as a stiffness optimization of a spatial parallel mechanism.

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Nomenclature

\[ \{B\} = \text{base coordinate frame} \]
\[ \{T\} = \text{tool coordinate frame} \]
\[ x_i = \text{position of center of platform based on } \{B\} \]
\[ p_i = \text{position of } i\text{th platform revolute joint based on } \{B\} \]
\[ B_i = \text{position of } i\text{th base revolute joint based on } \{B\} \]
\[ S_i = \text{position of } i\text{th revolute joint between links based on } \{B\} \]
\[ q_i = \text{angular position of } i\text{th joint } (i=1, \ldots, n) \]
\[ q_i = \text{angular position of actuation joints} \]
\[ q_i = \text{angular position of independent joints} \]
\[ l_{ij} = \text{length of } i\text{th link of } r\text{th link branch} \]
\[ d = \text{length of a side of platform} \]
\[ U = \text{relocation matrix reassigning the order of independent and dependent joint vector into a ascending order} \]
\[ V = \text{selection matrix extracting actuation joint vector among all joint vectors} \]
\[ g = \text{constraint equation between three revolute joints} \]

\[ J = \text{Jacobian between velocity of platform and that of all joints} \]
\[ G = \text{constraint Jacobian} \]
\[ \tau_i = \text{independent joint torques} \]
\[ \tau_i = \text{actuated joint torques} \]

Appendix

The \( L_{27}(1^{13}) \) and \( L_9(3^{3}) \) orthogonal array are presented in Tables 6 and 7, respectively.

References