PERGAMON

Mechanism and Machine Theory 37 (2002) 787–798

MECHANISM AND MACHINE THEORY

www.elsevier.com/locate/mechmt

Kinematic sensitivity analysis of the 3-UPU parallel mechanism

Chanhee Han, Jinwook Kim, Jongwon Kim, Frank Chongwoo Park *

School of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-742, South Korea

Received 28 February 2000; received in revised form 26 November 2001; accepted 26 November 2001

Abstract

This paper addresses the kinematic sensitivity of the three degree-of-freedom 3-UPU parallel mechanism, a mechanism consisting of a fixed base and a moving platform connected by three serial UPU chains. Although a mathematical mobility analysis confirms that the mechanism has three degrees of freedom, hardware prototypes exhibit unexpected large motions of the platform even when the three prismatic joints are locked at arbitrary configurations. Existing mathematical classifications of kinematic singularities also fail to explain the gross motions of the 3-UPU. This paper resolves this apparent paradox. We show that the 3-UPU is highly sensitive to certain minute clearances in the universal joint, and that a careful kinematic sensitivity analysis of the 3-UPU augmented with virtual joints satisfactorily explains the gross motions. Observations with a hardware experimental prototype confirm the results of our sensitivity analysis.

Keywords: Parallel mechanism; 3-UPU; Kinematic sensitivity; Kinematic singularity

1. Introduction

Despite their many advantages vis-à-vis serial mechanisms, classical 6-6 parallel mechanisms such as the Stewart platform suffer from a smaller workspace, complex mechanical design, and more difficult motion generation and control due to their complex kinematic analysis. In an attempt to overcome these and other limitations of 6-6 parallel mechanisms, many researchers have investigated various three and six degree-of-freedom 3-3 parallel mechanism designs as an alternative (e.g., [1,3,7]).

* Corresponding author. Tel.: +82-2-880-7133; fax: +82-2-883-1513.
E-mail address: fcp@haydn.snu.ac.kr (F.C. Park).

0094-114X/02/$ - see front matter
PII: S0094-114X(02)00021-6

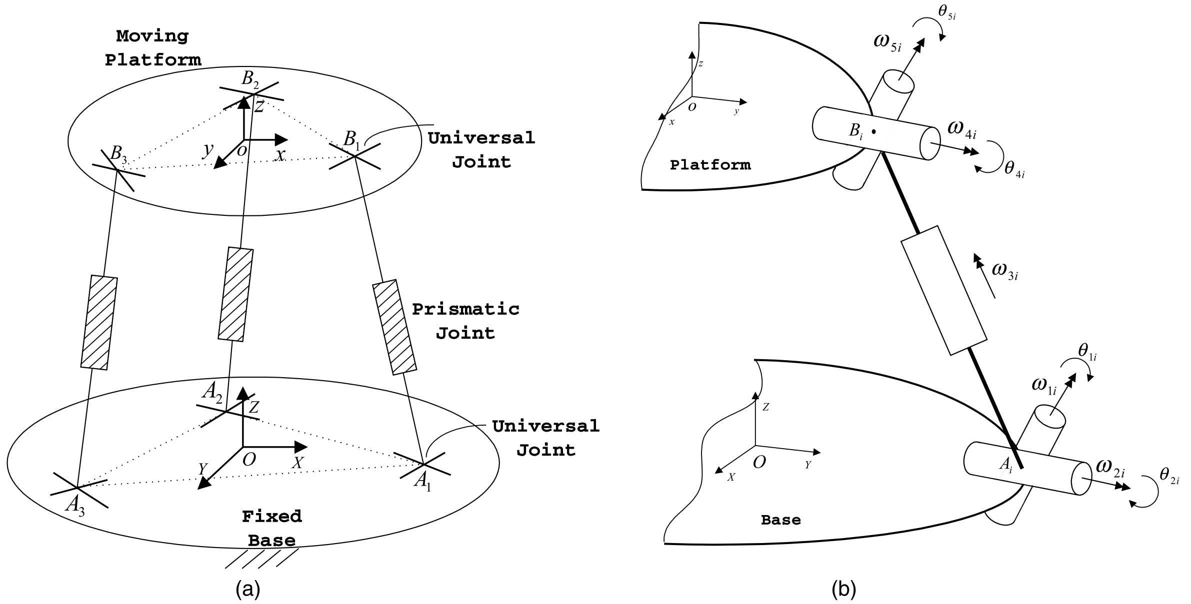
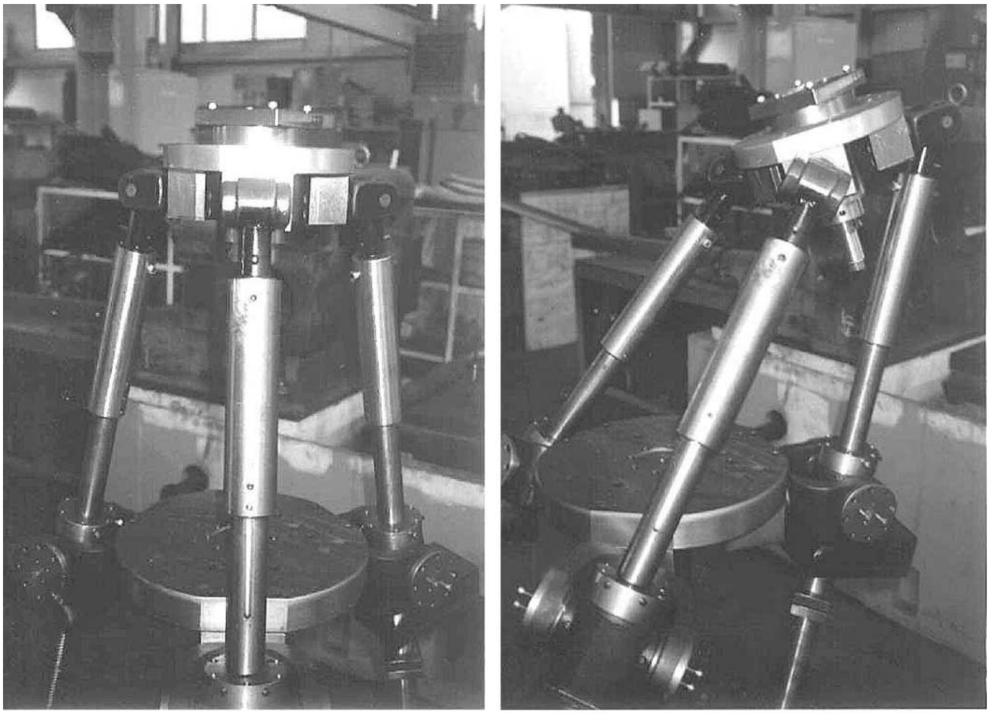


Fig. 1. The 3-UPU parallel mechanism: (a) the general 3-UPU mechanism; (b) the translational 3-UPU.

One of the more fascinating 3-3 designs is the three degree-of-freedom 3-UPU parallel mechanism; the basic structure of the mechanism is shown in Fig. 1. It consists of a fixed base and moving platform connected by three serial chains, with each chain having a universal–prismatic universal joint arranged in sequence. The universal joints are passive; only the three prismatic joints are actuated. In contrast to other 3-3 mechanisms, because the 3-UPU mechanism consists of only universal and prismatic joints, it is quite attractive from the manufacturing point of view. More interestingly, as first pointed out by Tsai [8], the universal joints can be attached in such a way that the moving platform only undergoes pure translational motion. Motivated by these results, Di Gregorio et al. [2,5] explore the conditions under which the more general 3-RRPRR mechanism (which includes the 3-UPU mechanism as a special case) can be arranged to undergo strictly translational motion.

Analysis of the kinematic constraint equations for both the general 3-UPU mechanism and the more general 3-RRPRR mechanism confirms that both have three degrees of freedom. Experiments with hardware prototypes of two representative 3-UPU designs, however, reveal an unexpected set of additional degrees of freedom—regardless of the platform configuration, when the prismatic joints are locked, the mechanism behaves as if it has additional degrees of freedom, rather than being a rigid structure as predicted by kinematic mobility analysis (see Fig. 2).

In this paper we first show that existing classifications of kinematic singularities fail to explain these redundant self-motions of the 3-UPU. We then show that this unexpected behavior can in fact be traced to minute clearances and manufacturing tolerances in each UPU assembly. Specifically, clearances in the bearing and shaft of each UPU assembly admit small torsional rotations about the leg axes, which in turn cause the gross motions of the moving platform. To show this we develop a more complete model that accounts for all possible infinitesimal motions of the



(a) (b)

Fig. 2. Hardware prototype of the SNU 3-UPU mechanism: (a) initial configuration, (b) redundant self-motion.

mechanism resulting from manufacturing errors or tolerances—each of the universal joints is augmented with a virtual revolute joint, effectively modeling the universal joint as a spherical joint. A first-order sensitivity analysis is then performed with the more complete kinematic model that resolves the apparent paradox.

The importance of identifying kinematic singularities of parallel mechanisms has long been recognized in the literature. Our study emphasizes the importance of kinematic sensitivity analysis, particularly when designing new parallel mechanisms. In this context careful attention must be paid to ensure that the kinematic models used for the kinematic sensitivity analysis reflect the complete range of design variations, clearances, and manufacturing tolerances that may occur; for parallel mechanisms containing universal joints it is particularly important to take into account torsional clearances of the type mentioned above. From this perspective the 3-UPU mechanism can be regarded as inherently more unstable than other three degree-of-freedom parallel mechanism architectures.

2. Singularity analysis

The general 3-UPU mechanism has three actuated prismatic joints in each limb, with two universal joints attached to the ends of each limb that connect the base platform and moving plate (see Fig. 1). As first observed by Tsai [8], if the joint axes are arranged to satisfy the following conditions, the 3-UPU mechanism will undergo purely translational motion:

1. The axes of the three revolute pairs embedded in the base and platform are coplanar and intersect at three points. The points form a triangle in the base that is similar to the one that the corresponding points form in the platform.
2. The axes of the two intermediate revolute pairs of each limb are parallel to each other and perpendicularly intersect the line of action of the prismatic actuator.

A 3-UPU mechanism satisfying the above conditions will be referred to as a translational 3-UPU mechanism. Noting the the 3-UPU consists of eight links, six universal joints, and three prismatic joints, application of Gruebler's formula confirms that the mechanism has three degrees of freedom.

We now find all the singular configurations of the translational 3-UPU mechanism. Before doing so we briefly review the classification of kinematic singularities as presented in [6]. Our presentation here is local coordinate-based in order to keep the mathematical technicalities from dominating; see [6] for a more complete coordinate-invariant geometric analysis of kinematic singularities for general mechanisms. Given an a degree-of-freedom mechanism consisting of a actuated joints and p passive joints, let $\theta_a \in \Re^a$ denote the vector of active joints, and $\theta_p \in \Re^p$ the vector of passive joints; [1] the combined joint vector θ is then defined by $\theta = (\theta_a, \theta_p)$. We further assume that each joint has one degree-of-freedom; a universal joint, for example, is therefore formed by connecting a pair of one degree-of-freedom revolute joints.

Express the kinematic closure constraints in the form $g(\theta_a, \theta_p) = 0$, where $g : \Re^a \times \Re^p \to \Re^p$. These equations suggest a view of the joint configuration space as an a-dimensional surface in $\Re^{a+p}$. Differentiating these constraint equations with respect to time, we get

$$[\, G_a(\theta) \quad G_p(\theta)\,] \begin{bmatrix} \dot{\theta}_a \\ \dot{\theta}_p \end{bmatrix} = 0. \tag{1}$$

If the $p \times (a+p)$ matrix

$$G(\theta) = [\, G_a(\theta) \quad G_p(\theta)\,], \tag{2}$$

is not of maximal rank at the given value of θ, we say that the mechanism is at a configuration space (or c-space) singularity. Geometrically c-space singularities correspond to, e.g., self-intersections of the joint configuration space manifold, and other points of the surface at which the tangent space changes dimension. For example, regarding the joint space of a planar four-bar linkage as a closed curve in $\Re^4$, the configuration space singularities occur at self-intersections of the curve. Clearly c-space singularities do not depend on which particular set of joints are actuated.

It is not difficult to see that if rank $(G(\theta)) < p$, then the square matrix G_p is singular (note however that the converse is not true). Configurations at which G_p becomes singular are referred

[1] Strictly speaking the joint space for the 3-UPU is defined by Cartesian products of flat tori and the real line, but since kinematic singularities are classified by their local differential properties, for our purposes it is sufficient to regard the joint space as a subset of Euclidean space.

to as actuator singularities. Assuming G_p^{-1} existed, one could write $\dot{\theta}_p = -G_p^{-1} G_a \dot{\theta}_a$; at an actuator singularity the passive joint rates cannot be determined from the active joint rates. Clearly the actuator singularities are determined by the choice of actuated joints. In fact, given a particular choice of actuated joints, the c-space singularities form a subset of the associated actuator singularities. One means of determining the c-space singularities is to find, for every possible combination of actuated joints, the corresponding set of actuator singularities, and taking the intersection of all these sets.

The final class of kinematic singularities correspond to configurations in which the end-effector loses a degree-of-freedom of motion, which is the standard notion of singularities for open chains. If $q \in \Re^a$ is any set of local coordinates for the joint space configuration manifold (e.g., one could choose $q = \theta_a$ over the range of values θ for which G_p^{-1} exists), then the forward kinematics can be written $f : \Re^a \to SE(3)$, $q \to f(q)$. Configurations q at which the forward kinematics Jacobian $J(q)$ becomes rank deficient are denoted end-effector singularities. Like c-space singularities, end-effector singularities also do not depend on the choice of actuated joints. We refer the reader to [6] for further examples and intuitive descriptions of each type of singularity.

We now analyze the singularities of both the translational 3-UPU mechanism, and a variant 3-UPU design obtained by switching the order of the joint axes in each of the universal joints (the exact geometry is described in more detail below). Our primary interest will be in c-space and actuator singularities, since only these singularities influence the internal mobility of the mechanism. Suppose a fixed and moving frame are attached to the centers of the fixed and moving platforms as shown in Fig. 3. The forward kinematics for each of the three UPU chains can then be written

$$g_i(\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{4i}, \theta_{5i}) = e^{A_{1i}\theta_{1i}} e^{A_{2i}\theta_{2i}} e^{A_{3i}\theta_{3i}} e^{A_{4i}\theta_{4i}} e^{A_{5i}\theta_{5i}} M, \quad i = 1, 2, 3. \tag{3}$$

Here we use the modern screw theoretic notation for the forward kinematics, in which the joint screw parameters are explicitly expressed as matrix exponentials; the A_i are elements of the Lie algebra SE(3), while $M \in SE(3)$ (see e.g., [4] for a review). For notational convenience define $\vec{\theta}_i = (\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{4i}, \theta_{5i}) \in \Re^5$, $i = 1, 2, 3$. Taking the right differential of the closure conditions

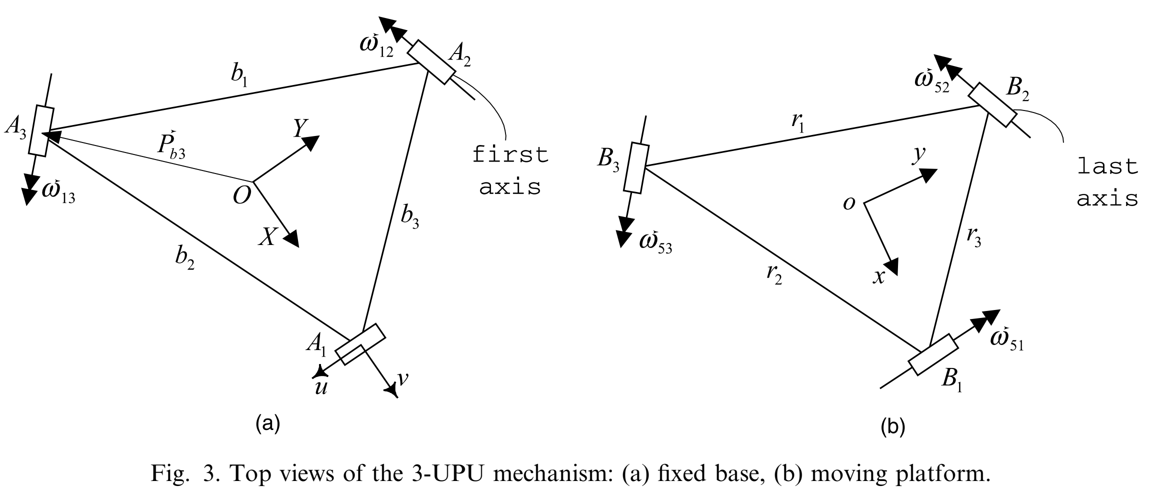


Fig. 3. Top views of the 3-UPU mechanism: (a) fixed base, (b) moving platform.

$g_1(\vec{\theta}_1) = g_2(\vec{\theta}_2)$ and $g_2(\vec{\theta}_2) = g_3(\vec{\theta}_3)$, we obtain the differential closure constraints $\dot{g}_1 g_1^{-1} = \dot{g}_2 g_2^{-1}$ and $\dot{g}_2 g_2^{-1} = \dot{g}_3 g_3^{-1}$, where each $\dot{g}_i g_i^{-1}$ can be expressed in matrix–vector form as

$$\begin{bmatrix} \omega_i \\ v_i \end{bmatrix} = \begin{bmatrix} \omega'_{1i} & \omega'_{2i} & \cdots & \omega'_{5i} \\ v'_{1i} & v'_{2i} & \cdots & v'_{5i} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{1i} \\ \vdots \\ \dot{\theta}_{5i} \end{bmatrix} = G_i(\vec{\theta}_i)\dot{\vec{\theta}}_i. \tag{4}$$

The columns (ω'_{ki}, v'_{ki}) are simply the infinitesimal screws (or twists) of the joints of the ith UPU chain expressed relative to the fixed frame. The differential closure constraints can now be combined in matrix form as follows:

$$\begin{bmatrix} G_1(\vec{\theta}_1) & -G_2(\vec{\theta}_2) & 0 \\ 0 & -G_2(\vec{\theta}_2) & G_3(\vec{\theta}_3) \end{bmatrix} \begin{bmatrix} \dot{\vec{\theta}}_1 \\ \dot{\vec{\theta}}_2 \\ \dot{\vec{\theta}}_3 \end{bmatrix} = G(\vec{\theta})\dot{\vec{\theta}} = 0. \tag{5}$$

C-space singularities occur at those configurations $\vec{\theta}$ for which $G(\vec{\theta})$ fails to be of maximal rank (12 in this case). Actuator singularities can be determined by removing columns 3, 8, and 13 (the columns corresponding to the actuated prismatic joints) in $G(\vec{\theta})$, and searching for configurations in which the resulting 12×12 matrix ($G_p(\vec{\theta})$ using our earlier notation) becomes rank deficient.

We now analyze the c-space singularities for the translational 3-UPU mechanism and the SNU 3-UPU mechanism. The latter is obtained by reversing the order of the universal joint axes (see Fig. 2). The kinematic parameters for the SNU and translational 3-UPU mechanisms are respectively listed in Tables 1 and 2. We first observe that the joint configuration space of the

Table 1
Kinematic parameters of the SNU 3-UPU

	ω	v
A_{11}	(1, 0, 0)	(0, 0, 0)
A_{21}	(0, 1, 0)	(0, 0, 259.8076)
A_{31}	(0, 0, 0)	(−0.1155, 0, 0.9933)
A_{41}	(0, 1, 0)	(−496.6555, 0, 202.0726)
A_{51}	(1, 0, 0)	(0, 496.6555, 0)
A_{12}	(−0.5000, 0.8660, 0)	(0, 0, 0)
A_{22}	(−0.8660, −0.5000, 0)	(0, 0, 259.8076)
A_{32}	(0, 0, 0)	(0.0577, −0.1000, 0.9933)
A_{42}	(−0.8660, −0.5000, 0)	(248.3277, −430.1163, 202.0726)
A_{52}	(−0.5000, 0.8660, 0)	(−430.1163, −248.3277, 0)
A_{13}	(−0.5000, −0.8660, 0)	(0, 0, 0)
A_{23}	(0.8660, −0.5000, 0)	(0, 0, 259.8076)
A_{33}	(0, 0, 0)	(0.0577, 0.1000, 0.9933)
A_{43}	(0.8660, −0.5000, 0)	(248.3277, 430.11163, 202.07261)
A_{53}	(−0.5000, −0.8660, 0)	(430.1163, −248.3277, 0)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 496.6555 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 2
Kinematic parameters of the translational 3-UPU

	ω	v
A_{11}	(0, 1, 0)	(0, 0, 259.8076)
A_{21}	(0.9933, 0, 0.1155)	(0, −30, 0)
A_{31}	(0, 0, 0)	(−0.1155, 0, 0.9933)
A_{41}	(0.9933, 0, 0.1155)	(0, 470, 0)
A_{51}	(0, 1, 0)	(−496.6555, 0, 202.0726)
A_{12}	(−0.8660, −0.5000, 0)	(0, 0, 259.8076)
A_{22}	(−0.4967, 0.8602, 0.1155)	(25.9808, 15, 0)
A_{32}	(0, 0, 0)	(0.0577, −0.1, 0.9933)
A_{42}	(−0.4967, 0.8602, 0.1155)	(−407.0319, −235, 0)
A_{52}	(−0.8660, −0.5000, 0)	(248.3277, −430.1163, 202.0726)
A_{13}	(0.8660, −0.5, 0)	(0, 0, 259.8076)
A_{23}	(−0.4967, −0.8602, 0.1155)	(−25.9808, 15, 0)
A_{33}	(0, 0, 0)	(0.0577, 0.1, 0.9933)
A_{43}	(−0.4967, −0.8602, 0.1155)	(407.0319, −235, 0)
A_{53}	(0.8660, −0.5, 0)	(248.3277, 430.1163, 202.0726)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 496.6555 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3-UPU can be viewed as a three-dimensional surface in the ambient space $\Re^{15}$. Self-intersections of this surface will correspond to c-space singularities. Moreover, for any given set of leg lengths, in general there will exist multiple solutions to the kinematic constraint equations.

With this geometric picture in mind, Fig. 4 plots the inverse of the condition number of $G(\vec{\theta})$ for the SNU 3-UPU mechanism. The x and y axes represent the lengths of the first and second legs of the mechanism, while the length of the third leg is fixed at an arbitrary value. At the home configuration, in which the three legs are of equal length, it is seen from the graph that $G(\vec{\theta})$ is rank-deficient, indicating a c-space singularity (and also an actuator singularity). The rank of $G(\vec{\theta})$ in fact drops by two at the home position. This is consistent with our geometric intuition obtained from the joint space surface picture, which suggests that the home position lies on a self-intersection of the joint space surface. Recall also that since the surface is three-dimensional, its self-intersection will be two-dimensional in the most generic case. Hence one would expect that the SNU 3-UPU would behave like a two-dimensional mechanism when all the legs are locked at equal lengths. This is consistent with the observed behavior in the prototype.

What the plot also shows is that for different sets of leg lengths, the mechanism clearly is not in a c-space singularity. Although we have plotted the graph for only one set of solutions—recall that for a given set of leg lengths, in general there are multiple solutions to the constraint equations, or equivalently for parallel mechanisms, multiple solutions to the forward kinematics equations—similar nonzero values of the inverse condition number are attained for other solutions. Experiments with the constructed prototype of Fig. 2, however, reveal that the mechanism exhibits gross motions even when the legs are locked at arbitrarily varying lengths away from

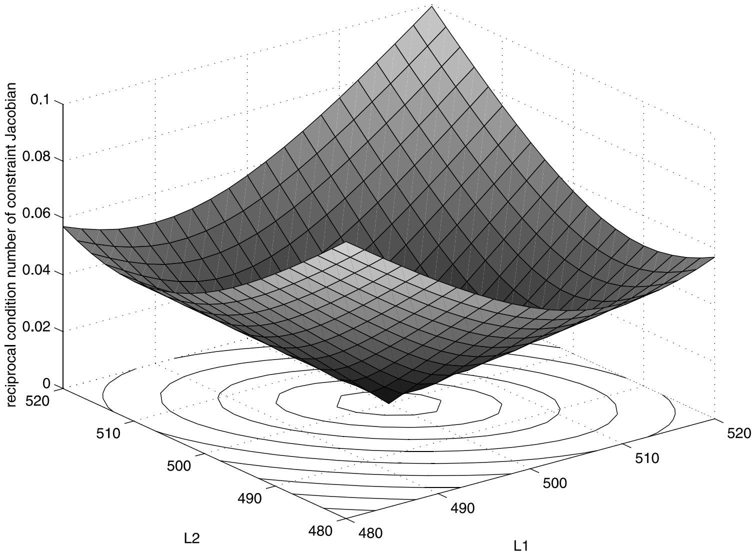


Fig. 4. Inverse condition number plot of the SNU 3-UPU.

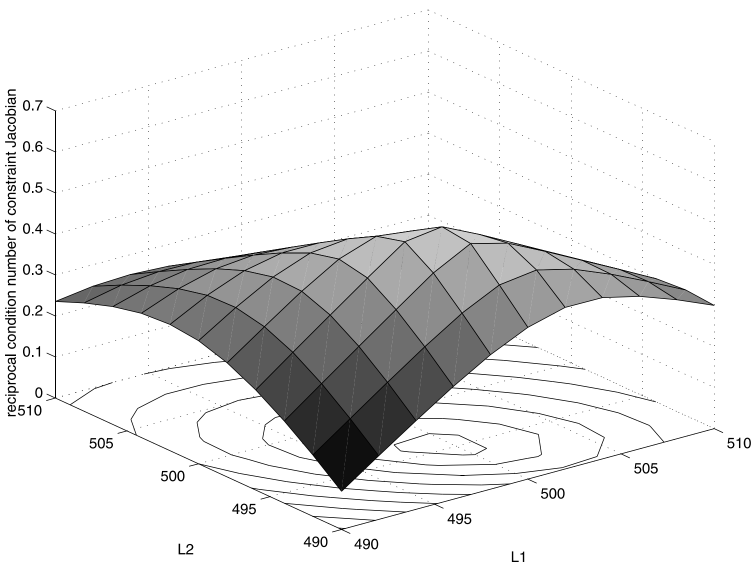


Fig. 5. Inverse condition number plot of the translational 3-UPU.

c-space and actuator singularities. Clearly singularities alone are insufficient for explaining these gross motions.

Fig. 5 shows a similar inverse condition number plot for the translational 3-UPU mechanism. Unlike the previous mechanism, the translational 3-UPU is neither in a c-space singularity nor actuator singularity at its home configuration. However, experiments with a prototype again reveal gross motions (albeit somewhat smaller in range than the SNU 3-UPU) that seemingly contradict the singularity analysis results.

3. Design sensitivity analysis

In this section we show that the extra degrees of freedom observed in the 3-UPU mechanism hardware prototypes can be traced to manufacturing tolerances and clearances in the serial limb assemblies. A closer examination of the prototypes reveals that the most likely source of clearances and loose tolerances is in the universal joint. Small manufacturing errors and clearances can allow for infinitesimal rotations about the legs, in particular clearances between the shaft and bearing (see Fig. 6). In this case the axis can move within a cone, and although the range of the motion is quite small—less than two degree—the consequences can be significant.

To test this hypothesis, we assume a virtual revolute joint is attached to each of the three upper universal joints, so that they emulate a spherical joint (see Fig. 7). Adopting the matrix exponential kinematic representation for screw motions once again, the forward kinematic equations for each of the serial UPU chains augmented with a virtual joint revolute can then be expressed in the form

$$g_i(\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{4i}, \theta_{5i}) = e^{A_{1i}\theta_{1i}} e^{A_{2i}\theta_{2i}} e^{A_{3i}\theta_{3i}} e^{V_i \epsilon_i} e^{A_{4i}\theta_{4i}} e^{A_{5i}\theta_{5i}} M, \quad i = 1, 2, 3. \tag{6}$$

Here $V_i = (\omega_{vi}, v_{vi}) \in SE(3)$ is the twist vector for the virtual revolute joint, and ϵ_i the corresponding rotation angle. Specifically, $\omega_{vi} = v_{3i}$ and $v_{vi} = -\omega_{vi} \times q_i$, with q_i the vector from the fixed frame origin to the center of the universal joint of chain i attached to the base, expressed in fixed frame coordinates.

As before, we construct the constraint Jacobian $G(\vec{\theta})$ by differentiating the closure conditions $g_1(\vec{\theta}_1) = g_2(\vec{\theta}_2)$ and $g_2(\vec{\theta}_2) = g_3(\vec{\theta}_3)$, where this time each $\vec{\theta}_i = (\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{vi}, \theta_{4i}, \theta_{5i})$, $i = 1, 2, 3$, is six-dimensional, and $g_i(\vec{\theta}_i)$ is defined as above. $G(\vec{\theta})$ therefore becomes a 12×18 matrix. For our sensitivity analysis we regard the virtual revolute joints as being actuated, and define the vector of actuated joints $\theta_a = (\theta_{31}, \theta_{v1}, \theta_{32}, \theta_{v2}, \theta_{33}, \theta_{v3}) \in \Re^6$. The 12×12 matrix $G_p(\vec{\theta})$ is then obtained by eliminating the six columns of $G(\vec{\theta})$ corresponding to the actuated joints (columns 3, 4, 9, 10, 15, and 16).

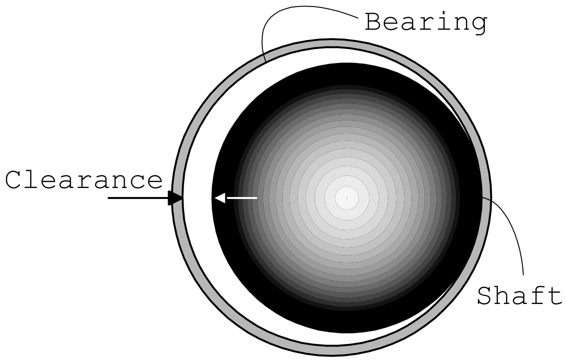


Fig. 6. Clearances in the shaft-bearing assembly.

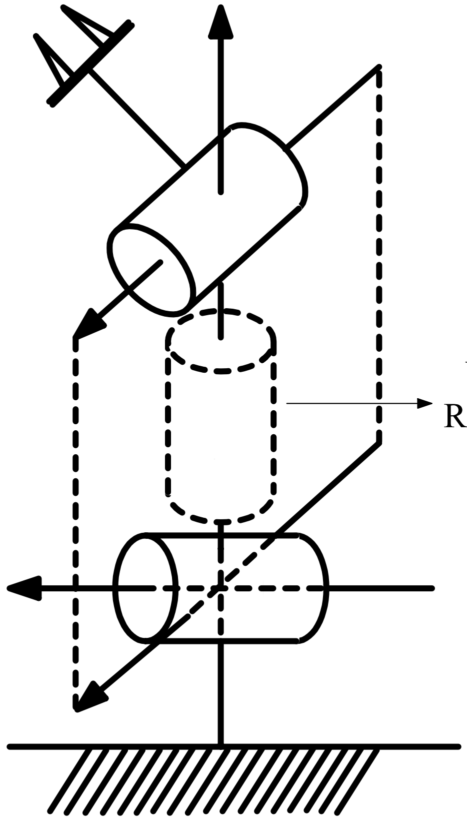


Fig. 7. The universal joint with a virtual revolute joint added.

We make two more definitions. Let $f : \Re^6 \rightarrow \Re^3$, $\theta_a \mapsto (x(\theta_a), y(\theta_a), z(\theta_a))$ denote the forward kinematic map from the actuated joints to the Cartesian x–y–z coordinates of the end-effector frame. Define the 3×3 matrix $J_v(\theta_a)$ to be the forward kinematics Jacobian of f with respect to the virtual revolute joints $(\theta_{v1}, \theta_{v2}, \theta_{v3})$, and the 3×3 matrix $J_l(\theta_a)$ to be the forward kinematics Jacobian of f with respect to the three actuated prismatic joints $(\theta_{31}, \theta_{32}, \theta_{33})$.

Table 3 shows the results of the sensitivity analysis for the SNU 3-UPU mechanism. For the analysis we set the length of leg three to a fixed 500 mm, and vary the lengths of the remaining two legs as shown. The first two columns show the inverse condition number μ of the matrices G and G_p at various configurations of the legs. As can be seen, these configurations are clearly neither c-space singularities nor actuator singularities.

Table 3
Sensitivity analysis results

L_1 (mm)	L_2 (mm)	$\mu(G)$	$\mu(G_p)$	$\sigma_{\max}(J_v)$	$\sigma_{\max}(J_l)$
505	510	0.114099	0.0652724	20.3475	0.997324
520	510	0.108077	0.0261158	34.6168	1.440720
530	510	0.108374	0.025207	36.0482	1.262301
530	520	0.106838	0.0261825	34.7962	1.450425

The third and fourth columns show the maximum singular values σ_{max} of the forward kinematics Jacobians J_v and J_l at the various leg configurations. Noting that the Jacobian J_v maps unit balls in the three-dimensional space of virtual revolute joints $(\theta_{v1}, \theta_{v2}, \theta_{v3})$ to ellipsoids in $\Re^3$, the singular values of J_v correspond to the absolute lengths of the principal axes of the ellipsoid. Choosing units of degrees for the virtual revolute joints and millimeters for the Cartesian position of the end-effector frame, the data imply that approximately one degree of motion in a single virtual revolute joint can result in roughly 20 mm of translational motion of the platform center. In constrast, examining the maximum singular values of J_l at the various configurations, we see that a 1 mm translation in one of the prismatic joints results in roughly 1 mm of translational motion of the platform center. Therefore one can conclude that roughly one degree of rotation of a virtual revolute joint is equivalent to 20 mm translation of a prismatic joint.

From a manufacturing standpoint, what the results suggest is that if the accuracy of the prismatic joint translation is expected to be <1 mm, then for a stable platform the clearances in the universal joint units should be such that the resulting torsional rotation about the legs should be $<0.05°$. Similar numbers are also obtained for the translational 3-UPU mechanism.

4. Conclusion

This paper has examined the causes of the gross self-motions observed in hardware prototypes of the 3-UPU parallel mechanism. After eliminating kinematic singularities as a primary cause of these motions, we show that the mechanism is extremely sensitive to small torsional rotations about the legs arising from clearances and manufacturing errors in the bearing-shaft assembly. A kinematic sensitivity analysis confirms that the suspected infinitesimal torsional rotations about the legs are indeed the source of the redundant self-motions.

One of the lessons to be drawn from this work is the importance of kinematic sensitivity analysis when designing parallel mechanisms. Our results indicate that some parallel mechanism designs are inherently more robust than others, and that certain designs are unstable. It is particularly important to verify that all possible design parameter variations, clearances, manufacturing and other assembly errors are accounted for in the kinematic model used for sensitivity analysis. Particularly in the case of universal joints, they should be augmented with additional virtual revolute joints to model torsional clearances.

Acknowledgements

This research was supported by the National Research Laboratory for Parallel Mechanisms, and by the BK21 Program in Mechanical Engineering, both at Seoul National University.

References

[1] G.R. Dunlop, T.P. Jones, Position analysis of a 3-DOF parallel manipulator, Mechanism and Machine Theory 32 (8) (1997) 903–920.

[2] R. Di Gregorio, V. Parenti-Castelli, A translational 3-degree of freedom parallel manipulator, in: J. Lenarcic, M.L. Husty (Eds.), Advances in Robot Kinematics: Analysis and Control, Kluwer, Dordrecht, 1998, pp. 49–58.

[3] J. Kim et al., Design and analysis of an overactuated parallel mechanism for rapid machining, IEEE Trans. Robotics & Autom. 17 (4) (2001) 423–434.
[4] R.M. Murray, Z. Li, S.S. Sastry, A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton, 1994.
[5] V. Parenti-Castelli, R. Di Gregorio, J. Lenarcic, Sensitivity to geometric parameter variation of a 3-degrees of freedom fully parallel manipulator, in: Proceedings of the JSME Int. Conf. Advanced Mechatronics, Okayama, Japan, 1998, pp. 364–389.
[6] F.C. Park, J.W. Kim, Singularity analysis of closed kinematic chains, ASME J. Mechanical Design 121 (1) (1999) 32–38.
[7] F. Sternheim, Computation of the direct and inverse kinematics model of the Delta 4 parallel robot, Robotersysteme 3 (1987) 199–203.
[8] L.W. Tsai, Kinematics of a three-degrees of freedom platform with three extensible limbs, in: J. Lenarcic, V. Parenti-Castelli (Eds.), Recent Advances in Robot Kinematics, Kluwer, Dordrecht, 1996, pp. 401–410.