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Kinematics-based Gait Planning of a Quadruped Gecko-like Model

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***Abstract*— Recent research on mobile robots is focused on locomotion in various environments. In this paper gait generation algorithm for a mobile robot that can locomote from ground to wall and climb vertical surfaces is proposed bio-inspired by a gecko lizard. The gait planning is based on the inverse-kinematics with the Jacobian of whole body, where the redundancy is solved by defining an object function to follow the real gecko posture and avoid collisions with the surface. The optimal scalar factor for these two objects is obtained by defining a superior object function that is to minimize the angular acceleration. The algorithm was verified through simulation of the gecko model locomoting given task paths avoiding abnormal joint movements and collisions.**

I. INTRODUCTION

Recently research on mobile robots that can locomote in various conditions such as irregular terrain [1], inclined/declined planes [2], stairways [3] etc has been active. Furthermore research on overcoming gravity to climb vertical walls [4, 5] has been a hot issue. However research to converge both studies that is to build a mobile robot that can overcome any ground conditions and moreover vertical walls hasn't been focused on.

The gecko lizard is a good model found in nature that can locomote in any conditions including vertical walls. The ability of gecko attaching to almost any surface using its directional adhesive hair has been mainly spotlighted but moreover this it has advantages such as a sprawled posture that decreases the falling moment for better climbing, and a flexible but still controllable waist that enables it to move freely from horizontal to vertical planes and vice versa. Inspiration from this biological example such as the concepts explained above seems to be an answer for the next generation mobile robots. However there haven't been any attempts of adopting these concepts to a mobile robot. So as the first step we propose a gait generation method for motion planning of a mobile robot that locomotes in any conditions, even vertical walls inspired by the gecko lizard.

There are several studies on gait planning of a quadruped robot via a static gait and a dynamic gait approach [6-9]. Among the two approaches, a gait planning via a static gait has been researched to be applied to overcome irregular terrains. However the studies cannot be directly applied to the gait planning based on the real gecko. Research on control of the waist and avoidance of collision during movement is necessary for the gecko to locomote from the ground to the wall or contra versa. However no research has been done previously in such matters.

We develop a gait generation method for straight and curved path planning on the ground, wall climbing and locomotion to inclined or declined planes based on static gait studies. First, a quadruped kinematic model with a waist joint-kinematic model of gecko which is shown in Fig. 1 is chosen and the kinematic analysis is done on it. The redundancy of the multi-DOF system is solved by defining an object function that includes a workspace centering and collision avoidance term. Optimization of the scalar factors of these two terms for good performance is also done by defining a superior object function that is designed to reduce the angular acceleration in joints. The detail procedure of gait planning is shown in Fig. 2. The footstep rule for gait generation is defined based on this progress and verified through simulation of locomotion in various conditions referred above.

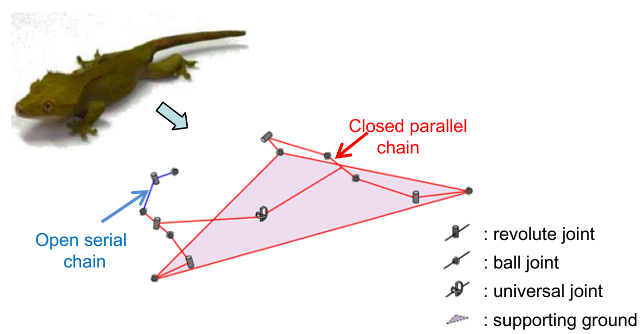

Fig. 1. Kinematic model of gecko(gecko model): The gecko model is based on the structure and joint movement of the 'New Caledonian crested gecko'. The gecko model consists of 11 links and 10 joints in total. The four legs are modeled as a SRS(Spherical-Revolute- Spherical) 7DOF linkage and the waist is a RU(Revolute-Universal) 3DOF linkage. The spherical joint which is the farthest joint from the body placed on the leg acts as a passive spherical joint. The number of the active joints is 19 in total(leg((7-3)×4=16DOF)+waist(3DOF)).

II. KINEMATICS

A. Assumptions of kinematic analysis

A quadruped animal needs at least three supporting points to avoid falling over while walking. We also assume no slip condition for a gecko due to its adhesiveness. Following this we assume that:

1. Gecko model always has at least three legs contacting the ground.
2. No slip condition at the feet

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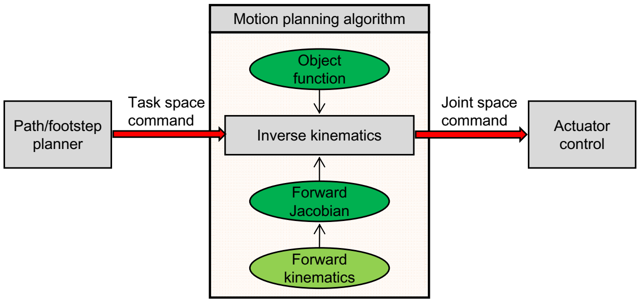


Fig. 2. Overview of motion planning algorithm.

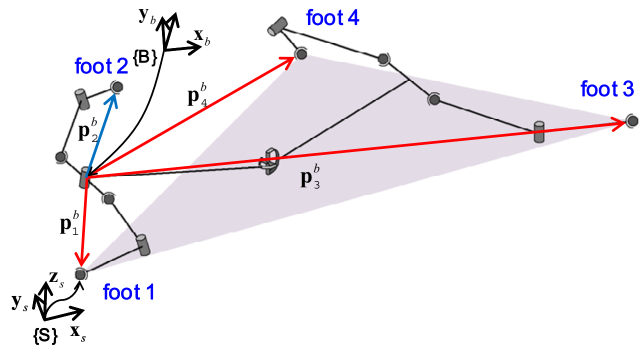


Fig. 3. Spatial frame and body frame on Gecko model.

B. Forward kinematics in spatial frame

The forward kinematics in body-frame can be derived straight forwardly by evaluating the $SE(3)$ matrix for each joint and multiplying them in order. The body-frame is attached to the gecko model and moves as time varies making it difficult to describe the movement in space. So we define the spatial frame which is a globally fixed frame.

The traditional method of solving the forward kinematics in spatial frame is to attach the frame to the ground and assume a leg to be a serial link based on the ground. It is solved by evaluating the angle between the ground and leg and angles at each joint. But since the gecko model (quadruped point-foot walker) has no actuator at the foot, the angle between the ground and leg is uncertain making it impossible to use the traditional method. So in this paper we introduce a new method to solve the forward kinematics in spatial frame with no need of evaluating the ground contact angle of the leg.

We first define the fixed legs and swinging leg. Here we choose foot 1(front left), foot 3(rear left), foot 4(rear right) to be fixed to the ground and foot 2(front right) to be the swinging foot. The following procedure holds for every other gait.

The three fixed feet are the only joints that never move during the gecko model moves. Since the triangle formed by the fixed feet is also always fixed unless the model is altered the spatial frame can be defined based on it. The frames are shown in Fig. 3. We choose any vertex as the origin of the spatial frame and define one side of the triangle that adjoins it to be the X-axis. The Z-axis is a unit vector normal to the triangle, and then the Y-axis is automatically defined. Although any point could be the origin we choose the vertex at the front fixed leg(foot 1: front left) to be the origin of the spatial frame.

$$\mathbf{x}_s^b = \frac{\mathbf{p}_3^b - \mathbf{p}_1^b}{|\mathbf{p}_3^b - \mathbf{p}_1^b|}, \quad \mathbf{z}_s^b = \frac{\mathbf{x}_s^b \times (\mathbf{p}_4^b - \mathbf{p}_3^b)}{|\mathbf{p}_4^b - \mathbf{p}_3^b|}, \quad \mathbf{y}_s^b = \mathbf{z}_s^b \times \mathbf{x}_s^b, \tag{1}$$

$$\mathbf{R}_{bs} = [\ \mathbf{x}_s^b \mid \mathbf{y}_s^b \mid \mathbf{z}_s^b\ ], \quad \mathbf{R}_{sb} = (\mathbf{R}_{bs})^\top, \tag{2}$$

$$\mathbf{T}_{sb} = \begin{bmatrix} \mathbf{R}_{sb} & -\mathbf{R}_{sb}\mathbf{p}_1^b \\ 0 & 1 \end{bmatrix}. \tag{3}$$

where $\mathbf{x}_s^b, \mathbf{y}_s^b, \mathbf{z}_s^b$ denote directional vectors from spatial-frame to body-frame, $\mathbf{p}_i^j$ denotes a directional vector of i^{th} foot in j-frame, and $\mathbf{R}_{ij}$ and $\mathbf{T}_{ij}$ denotes an $SO(3)$ and $SE(3)$ matrix from i-frame to j-frame, respectively.

However this triangle alters when the step is changed during walking. Since the spatial frame is defined based on the triangle at a step, in order to be a global frame we need a method to interpret the frames at each step w.r.t. the initial spatial frame.

At the instantaneous moment the swinging foot touches the ground all four feet are on the ground. The frame of the next step w.r.t. the current frame can be defined at this moment by evaluating the $SE(3)$ matrix between the two frames as follows:

$$\mathbf{T}_{s(i)s(i+1)} = \mathbf{T}_{s(i)b(i)}(\mathbf{T}_{s(i+1)b(i)})^{-1}. \tag{4}$$

By serially multiplying the matrices of (4) we can define any frame w.r.t. the initial spatial frame as follows:

$$\mathbf{T}_{s0bn} = \mathbf{T}_{s0s1}\mathbf{T}_{s1b1}(\mathbf{T}_{s2b1})^{-1}\cdots\mathbf{T}_{s(n-1)b(n-1)}(\mathbf{T}_{s(n)b(n-1)})^{-1}\mathbf{T}_{s(n)b(n)}. \tag{5}$$

By the definition of spatial frame the forward kinematics of foot 2(swing foot) w.r.t. spatial frame can be simply derived as follows:

$$\mathbf{p}_2^{s0} = \mathbf{T}_{s0bn}\mathbf{p}_2^{bn}. \tag{6}$$

C. Jacobian analysis

1) Constraint Jacobian: The closed parallel chain of the model has three dependant joints. The values of these joints are obtained from the constraint that three legs are fixed to the ground. Since the feet are fixed, the distance between each foot is constant as follows:

$$|p_i(\mathbf{q}_{up}, \mathbf{q}_v) - p_j(\mathbf{q}_{up}, \mathbf{q}_v)|^2 = g_i(\mathbf{q}_{up}, \mathbf{q}_v) = \text{const}, \tag{7}$$

where i, j=1,3,4 and $i \neq j$, p_i is the position of the i^{th} foot, $\mathbf{q}_{up}$ and $\mathbf{q}_v$ are the independent and dependant joints of the closed parallel chain respectively.

Deriving the constraint equation using (7) is complicated, so we use the partial derivative w.r.t. time. Differentiating g_i w.r.t. time, we can obtain

$$\frac{\partial g_i}{\partial \mathbf{q}_{up}}\frac{\partial \mathbf{q}_{up}}{\partial t} + \frac{\partial g_i}{\partial \mathbf{q}_v}\frac{\partial \mathbf{q}_v}{\partial t} = 0, \tag{8}$$

where, i=1, 3, 4. By defining $\mathbf{G}_{up,i} = \frac{\partial g_i}{\partial \mathbf{q}_{up}}$, the equation becomes:

$$\mathbf{G}_{up,i}\dot{\mathbf{q}}_u p + \mathbf{G}_{v,i}\dot{\mathbf{q}}_v = 0, \tag{9}$$

where i=1, 3, 4. Substituting $\mathbf{G}_{up} = [\mathbf{G}_{up,1}\ \mathbf{G}_{up,3}\ \mathbf{G}_{up,4}]^\top$ and $\mathbf{G}_v = [\mathbf{G}_{v,1}\ \mathbf{G}_{v,3}\ \mathbf{G}_{v,4}]^\top$, it becomes:

$$[\mathbf{G}_{up}\quad \mathbf{G}_v][\begin{array}{c}\dot{\mathbf{q}}_{up}\\ \dot{\mathbf{q}}_v\end{array}] = 0. \tag{10}$$

Since the inverse of $\mathbf{G}_v \in \mathbb{R}^{12\times 12}$ exists, we expand the equation and multiply each sides by $\mathbf{G}_v^{-1}$:

$$\dot{\mathbf{q}}_v = \mathbf{G}_v^{-1}\mathbf{G}_{up}\dot{\mathbf{q}}_{up} = \Phi[\dot{\mathbf{q}}_{up}|\dot{\mathbf{q}}_{us}] = \Phi\dot{\mathbf{q}}_u, \tag{11}$$

where, $\mathbf{q}_{us} \in \mathbb{R}^4$ is the joint value of the open serial chain and $\mathbf{q}_u \in \mathbb{R}^{16}$ is the independent joint value. To derive the constraint Jacobian, $\Phi \in \mathbb{R}^{3\times 16}$, between $\mathbf{q}_v$ and $\mathbf{q}_u$ in (12) we insert a zero column into the column of $-\mathbf{G}_v^{-1}\mathbf{G}_{up} \in \mathbb{R}^{3\times 12}$ that corresponds to $\mathbf{q}_{us}$. The constraint Jacobian is used to derive the forward Jacobian in the next section.

2) Forward Jacobian of whole body: The forward Jacobian of the sixteen independent joints can be obtained by differentiating (6) w.r.t. the independent joints($\mathbf{q}_u$) by definition:

$$\mathbf{J} = \mathbf{J}(\mathbf{q}_u) = \frac{\partial \mathbf{p}_2^s(\mathbf{q}_u, \mathbf{q}_v)}{\partial \mathbf{q}_u} = \frac{\partial \mathbf{p}_2^s(\mathbf{q}_u, \Phi\mathbf{q}_y)}{\partial \mathbf{q}_u} \in \mathbb{R}^{3\times 16}, \tag{12}$$

where, $\mathbf{p}_2^s$ is the swing foot position on spatial frame from (6), $\mathbf{q}_u$ is the independent joints, Φ is the constraint Jacobian and $\mathbf{J}(\mathbf{q}_u)$ is the forward Jacobian.

III. MOTION PLANNING BASED ON INVERSE KINEMATICS

A. Framework

The solution of the inverse kinematics for a redundant system is not unique. To find the optimum solution we use the null-space optimization method as in [11]:

$$\dot{\mathbf{q}} = \mathbf{J}^+\dot{\mathbf{X}} + \gamma(\mathbf{I} - \mathbf{J}^+\mathbf{J})\cdot\nabla_q H(\mathbf{q}), \tag{13}$$

where, $\mathbf{J}$ is the forward Jacobian of whole body from (12), $\mathbf{J}^+ = (\mathbf{J}^\top\mathbf{J})^{-1}\mathbf{J}^\top$is the Moore-Penrose pseudoinverse of $\mathbf{J}$, γ is a scalar factor and $H(\mathbf{q})$ is the object function. By projecting the gradient of the object function to the null-space of the forward Jacobian we obtain the solution that minimizes $H(\mathbf{q})$ while doing the task.

B. Definition of object function

We choose the following two conditions as criteria for an optimum motion.

1. A motion similar to the real gecko movement and that do not overstrain the joints: work space centering.

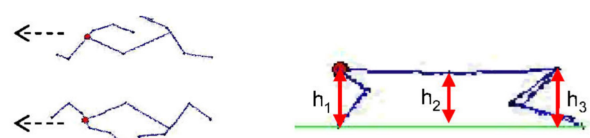
2. A motion that does not collide the ground or wall: collision avoidance.

These two conditions are defined as the object function:

$$\min H(\mathbf{q}) = \alpha\{\sum_{i=1}^{19}(q_i - q_{i,ref})^2\} + \beta\{\sum_{i=1}^{3}(h_i - h_{i,ref})^2\}. \tag{14}$$

The first term of the object function represents workspace centering. It is focused on minimizing the joint angle difference from the experimental values of a real gecko. Since the object function is to trace the pose of the real gecko movement a motion which is shown in Fig. 4 (a), similar to the real gecko is produced while doing a given task making the movement natural and helping to avoid impossible joint angles.

The second term is the collision avoidance term. By keeping the distance of the body (neck, waist, bottom) from the ground constant we avoid collision with the surface. The three distances to be keep constant is shown in Fig. 4 (b).



(a) Measured gecko's postures to be used as reference postures

(b) Distance between gecko's body and ground

Fig. 4. Object function reference terms. Left upper: 1/4 cycle step posture of real gecko (used as 2/4 cycle pose for gecko model). Left bottom: 3/4 cycle step posture of real gecko (used as 4/4 cycle pose for gecko model). Right: h_1, h_2, h_3 are heights from ground to neck, waist, bottom, respectively.

C. Selection of optimum scalar factor for object function

The scalar factor (α, β) in the object function (14) needs to be chosen since two object functions are linearly combined. Since the locomotion of the gecko model is dependant to (α, β), we use a method to obtain the optimum values.

First we define a function that is the total sum of the angular acceleration of the gecko model for a given path. The function concerns with values that should be minimized for better performance such as the required motor torque, system energy and the vibration produced. And since the values are proportional to the function we set it as a superior object function to be minimized and find the values of (α, β) that satisfy this:

$$\min f(\alpha, \beta) = \int_0^t (\sum_{i=1}^{19} |\ddot{q}_i(\alpha, \beta)|)dt \text{ along a predefined path.} \tag{15}$$

The optimum values of (α, β) are found using the gradient descent method and golden section search(method) [11, 12].

D. Generating swing trajectory of the foot

In each step, the gecko model first chooses the next footstep position and then swings its leg to it. In order to do this a swing trajectory needs to be generated.

We focus on the fact that the real gecko swings its foot outwards its body and adapt the same sing to the gecko model. We first create a new plane by rotating 90 degrees outward the plane that the current footstep point($\mathbf{p}_2^s(t_0)$), next footstep point($\mathbf{p}_2^s(t_0+t_{interval})$) and neck point($\mathbf{p}_{neck}^s$) is forming (Fig. 5). An isosceles triangle is made on this plane whose bottom side is $\mathbf{p}_2^s(t_0+t_{interval}) - \mathbf{p}_2^s(t_0)$ and height is d_{swing}. The vertex of the triangle is chosen to be the intermediate footstep position ($\mathbf{p}_2^s(t_0+0.5\times t_{interval})$). The swing trajectory of the foot is the spline created by the three points : the current footstep, intermediate footstep and the next footstep.

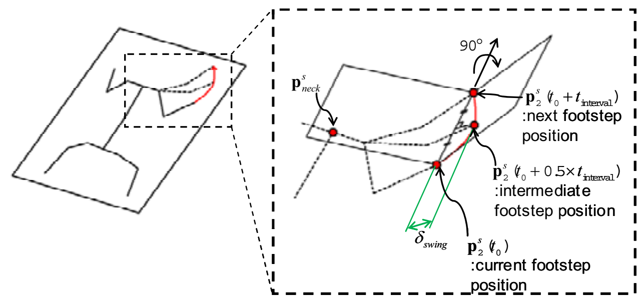


Fig. 5. Generation of swing foot trajectory.

E. Joint space control

In order to substitute the trajectory planning into the framework we evaluate the velocity at each time from the generated trajectory which is the task space command for discrete time dt:

$$\mathrm{d}\mathbf{p}_2^s = \mathbf{p}_2^s(t+\mathrm{d}t) - \mathbf{p}_2^s(t), \tag{16}$$

where, $\mathbf{p}_2^s(t)$ is the position of the foot. Substituting the object function and task into the framework we obtain the joint space command for discrete time dt:

$$\mathrm{d}\mathbf{q} = \mathbf{J}^{+}\mathrm{d}\mathbf{p}_2^s + \gamma(\mathbf{I}-\mathbf{J}^{+}\mathbf{J})\cdot\nabla_q H(\mathbf{q}), \tag{17}$$

where, $\mathbf{J}$ is the forward Jacobian of whole body, $\mathbf{J}^+$ is the Moore-Penrose pseudoinverse of $\mathbf{J}$ from (12), is the task space command from (16), $H(\mathbf{q})$ is the object function from (14).

By controlling the joint space with the solution in (17) the gecko model can obtain a minimum angular acceleration control input that satisfies the given task and minimizes the object function, simultaneously.

IV. SIMULATION IN VARIOUS MOVING CONDITIONS

A. Generation of footsteps

Some rules are set for the footstep generation of the walking gecko model. First, one stride should be the distance the gecko model travels during one cycle which is shown in Fig. 6. Next, the footstep position set for straight path should be symmetric every half cycle. In order to realize this, the following conditions should be satisfied:

1. The shape of the initial footstep position set is an isosceles trapezoid whose difference of the two base lengths is half of a stride.

2. One foot moves the length of one stride every 1/4 of a cycle.

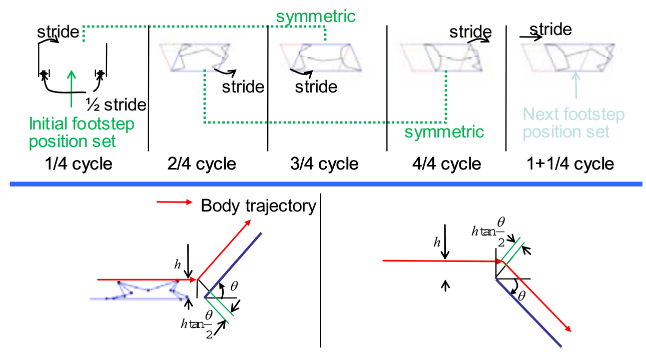


Fig. 6. Footstep rule. (Top: straight path footstep rule.) (Bottom: inclined/declined path footstep rule.)

The footstep position set for inclined/declined path requires foot step correction of $2h\tan(\frac{\theta}{2})$ due to the distance difference of the body and foot trajectory.

The straight path footstep, curved path footstep, inclined footstep, declined footstep can be generated based on this footstep rule. The straight path footstep is generated by shifting the initial footstep set one stride for each cycle(Fig. 7). The curved path footstep is generated by bending the initial footstep position of the straight path with a desired radius R and than rotating it one stride for each cycle w.r.t. the center of curvature. The inclined path footstep is generated by inserting $2h\tan(\frac{\theta}{2})$ of blank space into the straight path footstep where the inclining starts. Similarly the declined path footstep is generated by inserting footsteps for length $2h\tan(\frac{\theta}{2})$ into the straight path footstep where the declining starts.

B. Stability

Static stability of legged locomotion on a horizontal plane is determined by relation between positions of supporting feet and a position of the center of gravity(COG). To maintain static stability when walking on normal plane the COG should be located inside the closure of the three supporting legs. This is due to avoid the moment of the body making the robot loose stance and crash into the ground. However in the case of a gecko robot that has attachment force in the feet, the problem is slightly different. That is, the attachment force in the feet produces additional forces to overcome the pitching moment of the body.

The static stability during climbing vertical walls or ascending or descending inclined planes is guaranteed by two conditions. First the total attachment force should be larger than its body mass. Second the attachment force in the fore foot should be able to resist the pitch-black moment of the

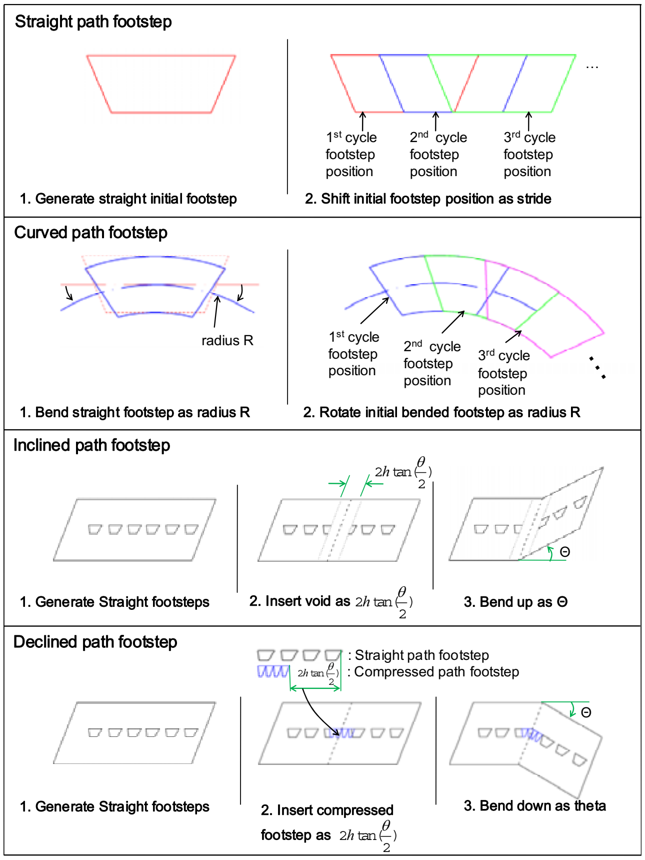


Fig. 7. Generation of straight, curved, inclined, declined path footsteps.

Fig. 8. Simulation result.

body which is critical in the gait the robot is changing one of its fore foot position, leaving only one fore foot left attached.

In both cases static stability could be obtained by producing enough attachment force in the feet respecting the body mass, especially the fore feet. Since the gait simulation presented in this paper is based on the simulation, the stability cannot be determined without assuming masses and attaching forces. Stability analysis result would give us required attaching forces of the feet for a specific robot to walk and climb stable.

C. Simulation results

The footstep generation was simulated using the gait planning algorithm. The simulation was carried out in normal PC computing environment. For a mobile robot in real time environment more consideration would be needed for computation cost due to limited computing ability. The test path included all conditions the gecko model could encounter such as straight/curved, inclined/declined paths. The scalar factors were optimized for the given path. The path for testing and result are shown in Fig. 8. Consequently, the gecko model movement is similar to that of the real gecko, avoiding abnormal joint movements and collisions with the ground. Smooth movement is generated with no vibration or jerking.

The locomotion from horizontal to vertical plane was checked to verify the performance of the collision avoidance term in the object function. The results with and without the avoidance term is shown in Fig. 9. Both simulations were consisted of the same footstep and swing foot trajectory, however the results varied depending on whether the neck, waist and hip were controlled or not. Collision with surface occurred with the absence of the collision avoidance term, however it was avoided with the term added.

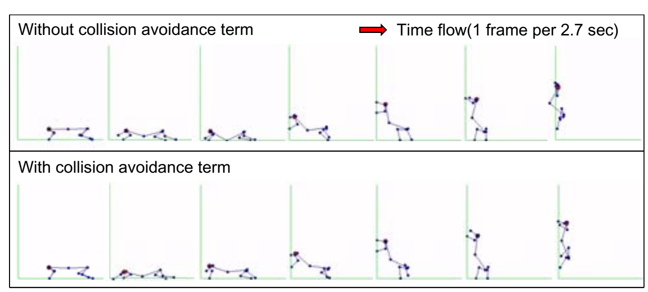


Fig. 9. Locomotion from horizontal to vertical plane of gecko model with and without collision avoidance term in object function.

The effect of scalar factor optimization was checked by comparing the simulation results before and after optimization. The angular acceleration was compared for a same given path which is shown in a 1^{st} slide of Fig. 8 and the resulting accelerations are shown in Fig. 10. The results without optimization(initial scalar factor used; (α, β) = 0.5/0.5) shows more fluctuation which results in more vibration and larger starting torque requirement. Also since the maximum acceleration is larger the maximum required torque increase which means a bigger actuator is needed. On the contrary, after optimization the motion is much smoother which means better durability and stability. The maximum joint angular acceleration was decreased as 36.43% and the sum of joint angular acceleration was decreased as 64.65%. And it is obvious that it is better for a mobile robot the smaller the actuator is, especially when climbing.

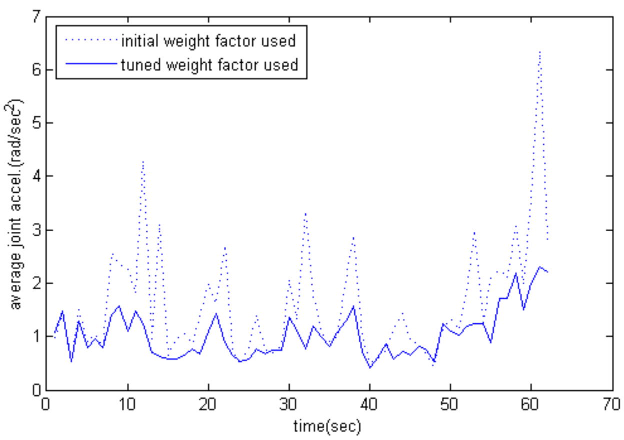


Fig. 10. Joint angular acceleration results with initial and optimal scalar factor.

V. CONCLUSIONS

This paper contains gait planning based on robotic analysis. First a spatial frame was defined that was global to any frame generated by three feet placed without considering the contact angle. The framework was divided into a serial and parallel part and was unified after analyzing separately. An object function was defined to solve uncertainty of the inverse kinematics of the redundant system. A workspace centering term based on real gecko movement and a collision avoidance term was defined in the object function for smooth and natural movement. A superior object function was defined to optimize the scalar factors in the object function. The gate generation based on swing foot trajectory and footstep generation rules was simulated and verified. Dynamic analysis and control for a gecko-inspired climbing robot is left for future work.

VI. ACKNOWLEDGMENTS

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