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Singularity Analysis of the *HALF* Parallel Manipulator with Revolute Actuators

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Abstract

*This paper concerns the singularity of the **HALF** parallel manipulator, a novel three degrees of freedom (DoFs) mechanism, with revolute actuators. The parallel manipulator, proposed previously, consists of a base plate, a movable platform, and three connecting legs. It has the advantages in terms of only single-DoF joints and high rotational capability. The parallel manipulator has wide application in the fields of industrial robots, simulators, parallel machine tools and any other manipulating devices that high rotational capability is required. The inverse kinematics problem and Jacobian matrices are recalled firstly. And then all possible singularities of the manipulator are analyzed in detail. The results of the paper are very useful for the apprehension and the application of the manipulator.*

1. INTRODUCTION

The last few years have witnessed an important development in the use of robots in the industrial world, mainly due to their flexibility. However, the mechanical architecture of the most common robots does not seem adapted to certain tasks. Other types of architectures have therefore recently been studied, and are being more and more regularly used within the industrial world [1~5]. This is so for the parallel manipulators. Parallel manipulators have many advantages over serial robots in terms of high load/weight ratio, velocity, stiffness, precision, and low inertia.

In the past two decades, some studies have led to the identification of several mechanical architectures with potential applications in parallel manipulators [6,7]. Most of the parallel mechanisms studied to date consist of six legs with six degrees of freedom. And these mechanisms are popular in the industrial applications, such as flight simulators, force/torque sensors and micro-motion manipulators, where the high load capability, multi-DoFs are needed. However, they suffer the problems of relatively small useful workspace and design difficulties. Furthermore, their direct kinematics is a very difficult problem. For such reason, spatial parallel mechanisms with less than 6 DoFs have increasingly attracted more and more researchers' attention [8,9,10,11,12] with respect to industry applications. Even then, these parallel mechanisms also suffer the problem of lower rotational capability, which limits their applications in some fields where high dexterity is needed, e.g., parallel kinematics machines [5]. This motivated us to design ***HALF*** parallel manipulators being with prismatic actuators [13] and revolute actuators [14]. The obvious advantage of the parallel manipulator is its high rotational capability, e.g., $\pm 45°$, because all involved joints are single-DoF ones.

In this paper, we focus our attention on the singularity analysis of the ***HALF*** parallel manipulator with revolute actuators. We attempt to find all possible configurations of the manipulator based on the singularity classification given by [15,16]. The results presented in this paper can be of great help in the design, application and control of such devices.

2. DESCRIPTION OF THE MANIPULATOR

2.1 Manipulator Structure

The ***HALF*** manipulator with revolute actuators and geometric parameters of the manipulator are shown in Fig.1 and Fig.2, respectively. The manipulator contains a triangular plate referred to as the moving platform. The platform is an isosceles triangle described by its parameter r, where $O'P_i = r$ ($i = 1, 2, 3$), as shown in Fig.2. The vertices of this platform are connected to a fixed base, which is also an isosceles triangle described by the parameter R and $OB_i = R$ ($i = 1, 2, 3$), through three legs, each of which consists of an upper link and a lower link. Two of the three legs have identical chains, in which the upper link is

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connected to the fixed base through a revolute joint and to the lower link with passive revolute joint. Each of the two lower links is then connected to the moving platform by a universal joint (or two revolute joints). The third leg is very different from the two ones. The upper link is connected to the fixed base through a revolute joint, and the lower link is actually composed of a planar four-bar parallelogram which is connected to the moving platform by another passive revolute joint. The two links are linked together by a revolute joint too. The movement of the platform can be obtained with the three input angular displacements θ_1, θ_2 and θ_3, which are given by three actuators, respectively.

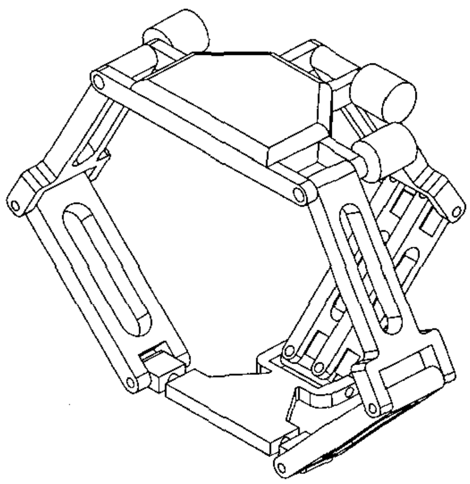

Fig.1 A ***HALF*** parallel manipulator with revolute actuators

2.2 Manipulator Capability

The first issue to address in the design of a mechanical system is to demonstrate its capability. As described by last Section, the proposed manipulator is a general manipulation device that must have three degrees of freedom when the input elements are active. Due to the arrangement of the links and joints, as shown in Figs.1 and 2, axes of the revolute joints in the first and second legs are parallel to each other. In such way, the two legs can provide two constraints on the moving platform with the rotation of about the $z-$axis and the translation along $x-$axis. Axes of the revolute joints connecting the fixed base, the upper link, and the lower link in the third leg are also parallel to each other. The third leg can provide two constraints on the rotation of the moving platform about z and x axes. Hence, the combination of the three legs constrains rotations of the moving platform with respect to x and z axes and the translation along $x-$axis. This leaves the mechanism with two translational degrees of freedom in $O-yz$ plane and one rotational degree of freedom about $y-$axis.

3. INVERSE KINEMATICS AND JACOBIAN MATRICES

3.1 Inverse kinematics

Mechanism kinematics deals with the study of the mechanism motion as constrained by the geometry of the links. The inverse kinematics problem involves mapping a known pose (position and orientation) of the output platform of the mechanism to a set of input joint variables that will achieve that pose.

A kinematics model of the manipulator is developed as shown in Fig.2. Vertices of the output platform are denoted as P_i ($i=1,2,3$), and vertices of the base platform are denoted as B_i ($i=1,2,3$). A fixed global reference system $\Re: O-xyz$ is located at the center of the side B_1B_2 with the $z-$axis normal to the base plate and the $y-$-axis directed along B_1B_2. Another reference frame, called the top frame $\Re': O'-x'y'z'$, is located at the center of the side P_1P_2. The $z'-$axis is perpendicular to the output platform and $y'-$axis directed along P_1P_2. Connecting joints between the upper and lower links are denoted as E_i. Lengths of upper and lower links for each leg are denoted as L_a and L_b, where $L_a = P_iE_i$ and $L_b = B_iE_i$ ($i=1,2,3$). What should be noted that, in some case, lengths of the links P_3E_3 and B_3E_3 can be different from that of P_1E_1 (P_2E_2) and B_1E_1 (B_2E_2).

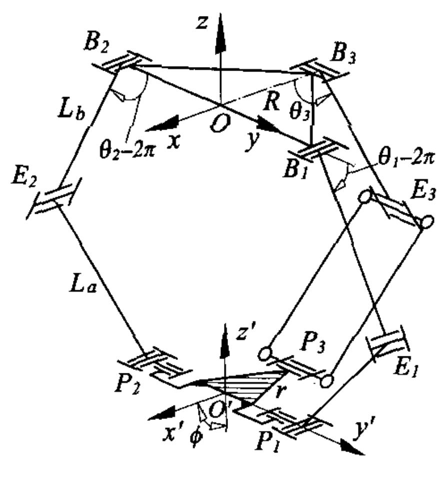

Fig.2 Geometric parameters of the manipulator

The objective of the inverse kinematics solution is to define a mapping from the pose of the output platform in a Cartesian space to the set of actuated inputs that achieve that pose. For this analysis, the pose of the moving platform is considered known, and the position is given by the position vector $(\boldsymbol{c})_{\Re}$ and the orientation is given by a matrix $\boldsymbol{Q}$. And there are

$$(\boldsymbol{c})_{\Re}=(0 \quad y \quad z)^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{Q}=\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \tag{2}$$

where the angle ϕ is the rotational degree of freedom of the output platform with respect to $y-$axis. The coordinate of the point P_i in the frame $\Re'$ can be described by the vector $(\boldsymbol{p}_i)_{\Re'}$ ($i=1,2,3$), and

$$(\boldsymbol{p}_1)_{\Re'}=\begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix},\ (\boldsymbol{p}_2)_{\Re'}=\begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix},\ (\boldsymbol{p}_3)_{\Re'}=\begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Then vectors $(\boldsymbol{p}_i)_{\Re}$ ($i=1,2,3$) in frame $O-xyz$ can be written as

$$(\boldsymbol{p}_i)_{\Re}=\boldsymbol{Q}(\boldsymbol{p}_i)_{\Re'}+(\boldsymbol{c})_{\Re} \tag{4}$$

Vectors $(\boldsymbol{e}_i)_{\Re}$ ($i=1,2,3$) will be defined as the position vectors of connecting joints E_i in frame $\Re$, and

$$(\boldsymbol{e}_1)_{\Re}=\begin{pmatrix} 0 \\ R+L_b\cos\theta_1 \\ L_b\sin\theta_1 \end{pmatrix},\ (\boldsymbol{e}_2)_{\Re}=\begin{pmatrix} 0 \\ L_b\cos\theta_2-R \\ L_b\sin\theta_2 \end{pmatrix},$$

$$(\boldsymbol{e}_3)_{\Re}=\begin{pmatrix} L_b\cos\theta_3-R \\ 0 \\ L_b\sin\theta_3 \end{pmatrix} \tag{5}$$

Then the inverse kinematics of the parallel manipulator can be solved by writing following constraint equation

$$\left\|[\boldsymbol{p}_i-\boldsymbol{e}_i]_{\Re}\right\|=L_a,\quad i=1,2,3 \tag{6}$$

from which one obtains

$$(r+y-R-L_b\cos\theta_1)^2+(z-L_b\sin\theta_1)^2=L_a^2 \tag{7}$$

$$(y-r+R-L_b\cos\theta_2)^2+(z-L_b\sin\theta_2)^2=L_a^2 \tag{8}$$

$$(r\cos\phi-R+L_b\cos\theta_3)^2+y^2+(r\sin\phi+z-L_b\sin\theta_3)^2=L_a^2 \tag{9}$$

and

$$A_is_i^2+B_is_i+C_i=0 \tag{10}$$

in which

$$s_i=\tan(\theta_i/2)$$
$$A_1=(r+y-R)^2+z^2+L_b^2-L_a^2+2(r+y-R)L_b$$
$$B_1=-4zL_b$$
$$C_1=(r+y-R)^2+z^2+L_b^2-L_a^2-2(r+y-R)L_b$$
$$A_2=(R+y-r)^2+z^2+L_b^2-L_a^2+2(R+y-r)L_b$$
$$B_2=B_1$$
$$C_2=(R+y-r)^2+z^2+L_b^2-L_a^2-2(R+y-r)L_b$$
$$A_3=(r\cos\phi-R)^2+y^2+L_b^2+(z+r\sin\phi)^2-L_a^2-2(r\cos\phi-R)L_b$$
$$B_3=-4(z+r\sin\phi)L_b$$
$$C_3=(r\cos\phi-R)^2+y^2+L_b^2+(z+r\sin\phi)^2-L_a^2+2(r\cos\phi-R)L_b$$

Hence, for a given manipulator and prescribed values of the position and orientation of the moving platform, the required actuator inputs can be directly computed as

$$\theta_i=2\tan^{-1}(s_i) \tag{11}$$

where

$$s_i=\left(-B_i\pm\sqrt{B_i^2-4A_iC_i}\right)/(2A_i) \tag{12}$$

From Eq.(12), we can see that there are eight inverse kinematics solutions for a given pose of the parallel manipulator. To obtain the inverse configuration as shown in Fig.1, the sign "$\pm$" in Eq.(12) should be "+" for $i=1$ and "−" for $i=2,3$.

3.2 Jacobian matrices

Equations (7)~(9) can be differentiated with respect to time to obtain the velocity equations, which leads to

$$L_b[z\cos\theta_1-(r+y-R)\sin\theta_1]\dot{\theta}_1=(r+y-R-L_b\cos\theta_1)\dot{y}+(z-L_b\sin\theta_1)\dot{z} \tag{13}$$

$$L_b[z\cos\theta_2-(R+y-r)\sin\theta_2]\dot{\theta}_2=(R+y-r-L_b\cos\theta_2)\dot{y}+(z-L_b\sin\theta_2)\dot{z} \tag{14}$$

$$L_b[(z+r\sin\phi)\cos\theta_3+(r\cos\phi-R)\sin\theta_3]\dot{\theta}_3=y\dot{y}+(r\sin\phi+z-L_b\sin\theta_3)\dot{z}+r[(z-L_b\sin\theta_3)\cos\phi+(R-L_b\cos\theta_3)\sin\phi]\dot{\phi} \tag{15}$$

Rearranging Eqs.(13)~(15) leads to an equation of the form

$$\boldsymbol{A}\dot{\boldsymbol{\theta}}=\boldsymbol{B}\dot{\boldsymbol{p}} \tag{16}$$

where $\dot{\boldsymbol{p}}$ is the vector of output velocities defined as

$$\dot{\boldsymbol{p}}=(\dot{y} \quad \dot{z} \quad \dot{\phi})^{\mathrm{T}} \tag{17}$$

and $\dot{\boldsymbol{\theta}}$ is the vector of input velocities defined as

$$\dot{\boldsymbol{\theta}}=(\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3)^{\mathrm{T}} \tag{18}$$

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are, respectively, the 3×3 inverse and forward Jacobian matrices of the manipulator and can be expressed as

$$\boldsymbol{A}=diag(a_{11},\ a_{22},\ a_{33}) \tag{19}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & 0 \\ y & b_{32} & b_{33} \end{bmatrix} \quad (20)$$

in which

$a_{11} = L_b[z\cos\theta_1 - (r+y-R)\sin\theta_1]$

$a_{22} = L_b[z\cos\theta_2 - (R+y-r)\sin\theta_2]$

$a_{33} = L_b[(z+r\sin\phi)\cos\theta_3 + (r\cos\phi - R)\sin\theta_3]$

$b_{11} = r+y-R-L_b\cos\theta_1$

$b_{12} = z - L_b\sin\theta_1$

$b_{21} = R+y-r-L_b\cos\theta_2$

$b_{22} = z - L_b\sin\theta_2$

$b_{32} = r\sin\phi + z - L_b\sin\theta_3$

$b_{33} = r[(z-L_b\sin\theta_3)\cos\phi + (R-L_b\cos\theta_3)\sin\phi]$

The Jacobian matrix of the parallel manipulator can be written as

$$J = A^{-1}B \quad (21)$$

4. SINGULARITY ANALYSIS

In the parallel manipulator, singularities occur whenever A, B or both, become singular. Because singularity leads to a loss of the controllability and degradation of the natural stiffness of manipulators, the analysis of parallel manipulators has drawn considerable attention [15, 16]. Based on the forward and inverse Jacobian matrices, a classification of the singularities pertaining to parallel manipulators into three main groups was suggested [16].

4.1 The firs kind of singularity

The first kind of singularity occurs when A becomes singular but B is invertible, i.e.

$$\det(A) = 0 \quad \text{and} \quad \det(B) \neq 0 \quad (22)$$

This kind of singularity corresponds to the configuration in which the chain reaches either a boundary of its workspace or an internal boundary limiting different subregions of the workspace where the number of branches is not the same. This condition is encountered here when one of the diagonal entries of A vanishes, that is

$$z\cos\theta_1 - (r+y-R)\sin\theta_1 = 0 \quad (23)$$

or

$$z\cos\theta_2 - (R+y-r)\sin\theta_2 = 0 \quad (24)$$

or

$$(z+r\sin\phi)\cos\theta_3 + (r\cos\phi - R)\sin\theta_3 = 0 \quad (25)$$

which mean that, this type of configuration for our manipulator is reached whenever the lower and upper links for each one of the three legs $P_1E_1B_1$, $P_2E_2B_2$ and $P_3E_3B_3$ are completely extended or folded. One of such singularity is shown in Fig.3.

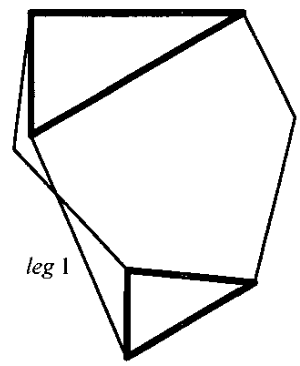

Fig.3 One singular configuration of the first kind of singularity: leg 1 is completely extended

4.2 Second kind of singularity

The second kind of singularity, occurring only in closed kinematics chains, arises when B becomes singular but A is invertible, i.e., when

$$\det(A) \neq 0 \quad \text{and} \quad \det(B) = 0 \quad (26)$$

In such configuration, the output link is locally movable even when all the actuated joints are locked, and the output link cannot resist one or more forces or moments even when all actuators are locked.

From Eq.(20), one obtains

$$\det(B) = b_{33}(b_{11}b_{22} - b_{12}b_{21}) \quad (27)$$

from which, one can see that $\det(B) = 0$ can be satisfied if (a) $b_{33} = 0$; (b) $b_{21} = b_{11} = 0$; (c) $b_{22} = b_{12} = 0$ and (d) both $b_{22} = b_{12}$ and $b_{11} = b_{21}$. The manipulator will be in the configuration that lower link E_3P_3 of the third leg $P_3E_3B_3$ is in the plane defined by P_1, P_2 (in this paper, it is called the moving platform plane) and P_3 if $b_{33} = 0$, the configuration that $E_1P_1 \perp P_1P_2$ if $b_{11} = 0$, $E_2P_2 \perp P_1P_2$ if $b_{21} = 0$. When $b_{12} = 0$ or $b_{22} = 0$, $E_1P_1P_2$ or $P_1P_2E_2$ will be completely extended or folded. From Eq.(20), one can also find that if $b_{12} = b_{11} = 0$ or $b_{22} = b_{21} = 0$, the manipulator can be in a singular configuration as well. But $b_{11} = 0$ and $b_{12} = 0$ correspond to different configurations of the first leg. Because a leg cannot be in two different configurations at the same time, the cases $b_{12} = b_{11} = 0$ and $b_{22} = b_{21} = 0$ will not occur actually.

If $b_{22} = b_{12}$, i.e., $z - L_b\sin\theta_1 = z - L_b\sin\theta_2$, there is

$$\theta_2 = \theta_1 \ \text{or} \ \theta_2 = \pi - \theta_1, \quad (\theta_1,\ \theta_2 \in [0,\ 2\pi]) \quad (28)$$

Substitution into $b_{11} = b_{21}$ leads to

$$R = r \text{ or } R + L_b \cos\theta_1 = r \quad (29)$$

From above analysis, we can see that the second kind of singularity for the manipulator will arise if any one of the following cases occurs:

- The lower link E_3P_3 of the third leg $P_3E_3B_3$ is in the moving platform plane the manipulator will be in the second kind of singularity, as shown in Fig.4(a);
- $E_1P_1 \perp P_1P_2$ and $E_2P_2 \perp P_1P_2$, as shown in Fig.4(b);
- $E_1P_1P_2$ and $P_1P_2E_2$ are completely extended or folded, i.e., four points E_1, P_1, E_2, P_2 are collinear, as shown in Fig.4(c);
- $R = r$ and $\theta_2 = \theta_1$, as shown in Fig.4(d);
- $R + L_b \cos\theta_1 = r$ and $\theta_2 = \pi - \theta_1$, as shown in Fig.4(e);
- $r = 0$, which leads to $b_{33} = 0$.

from which one can see that the singularities as shown in Figs.4 (b) and (d) are different from others, because they correspond to a different assembling mode, i.e., the assembling mode as shown in Fig.1 cannot reach such singular configurations.

4.3 The third kind of singularity

The third kind of singularity occurs when both $\boldsymbol{A}$ and $\boldsymbol{B}$ become simultaneously singular. This singularity is of a slightly different nature than the first two since it is not only configuration- but also architecture-dependent [15]. This corresponds to configurations in which the chain can undergo finite motions when its actuators are locked.

From the analysis in Section 4.2, we can see if $R = r$ and $\theta_2 = \theta_1$, the determinant of matrix $\boldsymbol{B}$ will be zero. In the geometric condition $R = r$, from Eq.(19), $\det(\boldsymbol{A})$ can also equal to zero at point $(y = 0,\ z = 0)$. Then $R = r$ and $\theta_2 = \theta_1$ with $(y = 0,\ z = 0)$ are the architecture and configuration conditions, respectively, for the third kind of singularity.

Additionally, due to the architecture of the manipulator, as shown in Fig.1, when the relationship between the parameters is

$$R = r + L_a + L_b \quad (30)$$

or

$$r = R + L_a + L_b \quad (31)$$

the manipulator can be assembled but cannot move any more, this is the so-called architecture singularity.

Generally, all possible singular configurations of a parallel manipulator with three DoFs can be listed easily, which is one of the reasons that such manipulators can be applied to industrial applications as quickly as possible. As shown in this Section, the singularity of ***HALF*** parallel manipulators with revolute actuators can be roughly classified into eleven cases, including one case of architecture singularity that $R = r + L_a + L_b$. From the view of singularity, a ***HALF*** parallel manipulator with prismatic actuators is better than that with revolute actuators. In the manipulator with prismatic actuators, the second kind of singularity occurs only in the cases of $R = r$ and the third leg being in its moving platform plane.

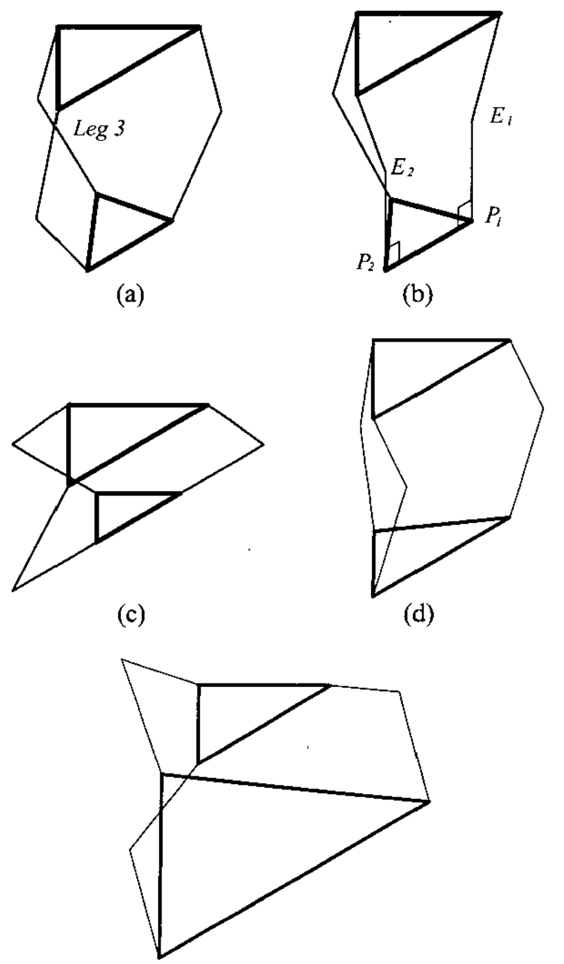

Fig.4 Some singular configurations of the second kind of singularity: (a) the lower link of the leg 3 is in the moving platform plane; (b) $E_1P_1 \perp P_1P_2$ and $E_2P_2 \perp P_1P_2$; (c) lower links of the first and second legs are collinear; (d) $R = r$ and $\theta_2 = \theta_1$; (e) $R + L_b \cos\theta_1 = r$ and $\theta_2 = \pi - \theta_1$.

5. CONCLUSION

One of the reasons that parallel manipulators with less than six DoFs are popular in applications is that all possible singularities can be found out. This paper concerns the singularity of a new spatial 3-DoF parallel manipulator, the ***HALF*** parallel manipulator with revolute actuators, which is an interesting mechanism with high rotational capability. Generally, three kinds of singularities for parallel manipulators classified by former researchers previously are advanced to the investigation of the ***HALF*** parallel manipulator. All possible singular configurations are presented, which show that there are eleven cases to arise the singularities, including one case of architecture singularity. The obtained results are very useful for the apprehension and the application of the manipulator.

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