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A three translational DoFs parallel cube-manipulator

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SUMMARY

This paper concerns the presentation and analysis of a type of three translational degrees of freedom (DoFs) parallel cube-manipulator. The parallel manipulators are the topology architectures of the DELTA robot and Tsai's manipulator, respectively, which have three translational DoFs. In the design, the three actuators are arranged according to the Cartesian coordinate system, which means that the actuating directions are normal to each other, and the joints connecting to the moving platform are located on three sides of a cube, for such reason we call this type of manipulator the parallel cube-manipulator. The kinematics problems, singularity, workspace, compliance characteristic of the manipulator are investigated in the paper. The analysis results show that the manipulators have the advantages of no singularities in the workspace, relatively more simple forward kinematics, and existence of a compliance center. The parallel cube-manipulator can be applied to the fields of micro-motion manipulators, remote center compliance (RCC) devices, assembly, and so on.

KEYWORDS: Parallel manipulators; Singular configurations; Compliance; Kinematics; Workspace.

1. INTRODUCTION

After a motion simulator with parallel kinematics chains was introduced in 1965,[1] parallel manipulators received more and more attention because of high stiffness, high speed, high accuracy, compact and high carrying capability. And more and more parallel manipulators with specified number and type of degrees of freedom (DoFs) have been proposed. Behi[2] described a 6-DoF configuration with three legs where each leg consists of a PRPS chain. Hudgens and Tesar[3] investigated a device with six inextensible legs where each leg is driven by a four-bar mechanism mounted on the base. Romiti and Sorli[4] introduced a 6-DoF parallel manipulator named TuPaMan with three legs, each of which is composed of a double parallelogram, one sliding joint, the connecting bar, and a ball joint. Austad[5] mentioned a hybrid architecture with 5 DoFs based on two parallel mechanisms. Pierrot and Company[6] presented a new family of parallel manipulators with 4 DoFs, which are 3 translations and 1 rotation. Kim and his colleague[7] proposed a 6-DoF parallel mechanism named Eclipse-II, which has the advantage of enabling continuous 360-degree spinning of the platform based on redundant actuators. And Liu *et al.*[8] proposed a spatial parallel manipulator with two translational DoFs and one rotational DoF, which has high mobility. They have been applied widely to the fields of motion simulator, force/torque sensor, packaging, micro-motion manipulators, medical devices, compliance devices and machine tools. In these designs, parallel manipulators with three translational degrees of freedom have been playing important roles in the industrial applications,[9–11] especially, the DELTA robot,[9] which is covered by a family of 36 patents.[12]

Generally, most of these manipulators suffer from both singular configurations within the workspace and multiple solutions of the direct geometric model. The objective of the paper is to design a three translational DoFs parallel manipulator which avoids these drawbacks and which can be well adapted to the applications of micromotion manipulators, remote center compliance (RCC) devices, and precision assembly machines. In the design, the three actuators are arranged according to the Cartesian coordinate system, which means that the actuating directions are normal to each other, and the joints connecting to the moving platform are located on three planes being perpendicular to each other too; hence, we call this type of manipulator a parallel cube-manipulator. The kinematics problems, singularity, workspace and compliance characteristics of the manipulator are investigated in the paper. The kinematics analysis of the manipulator shows that the solution for the inverse and forward kinematics is unique for the reason of interference. The manipulators have the advantages of high compactness and stiffness, no singularities in the workspace, relatively more simple forward kinematics, and existence of a compliance center. Other advantages of this type of parallel manipulator will be investigated in the future work.

2. PROPOSAL OF THE PARALLEL CUBE-MANIPULATOR

In the field of parallel manipulators, an interesting problem is to find a method to design a mechanical architecture for a parallel manipulator being given its number and type of degree of freedom. After Gough established the basic principles of a mechanism with a closed-loop kinematics structure in 1947, many other parallel manipulators with specified number and type of degrees of freedom have also been proposed. Fig. 1 is the general architecture of 6-DoF parallel manipulators. Theoretically speaking, the arrangement of the six legs of the manipulator could be at will,

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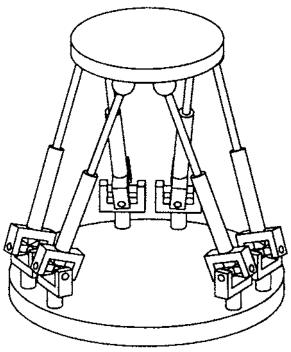

Fig. 1. A general 6-DoF parallel manipulator.

which will lead to some potential 6-DoF parallel manipulators, such as a manipulator as shown in Fig. 2, where the legs are arranged as 3-2-1 style. This architecture has the application advantage in micro system.[13] The arrangement disposal of six legs, as shown in Fig. 3, will make the manipulator move freely along a specified direction, which is very useful for the application in industry.[14]

Letting every two legs of the six legs of the manipulator, as shown in Fig. 1 be parallel to each other will lead to a 6-DoF parallel manipulator similar to that with revolute lactuators presented by Pierrot shown in Fig. 4.[15] The number of DoFs of the manipulator will be different if inputs of the two parallel legs are identical. This is actually a topology mechanism of the DELTA with linear actuators. Similarly, the manipulator shown in Fig. 4 can also be evolved into the DELTA robot. The fact that the outputs of two nearby revolute actuators in Fig. 4 are always same to each other will result in the different output of the moving platform. Actually, this is the design of the well-known DELTA robot, as shown in Fig. 5.

Apart from the DELTA robot, there are also many other parallel manipulators with three DoFs. For example, the spatial 3-DoF 3-RPS parallel manipulator presented by Hunt in the early of 80's.[16] The output of the manipulator is the complex motion, which are one translation and two rotations with continuously changed axes.[17] For such reason, the manipulator has less good applications except for the micro-motion device.[18] The similar manipulator includes CaPaMan in Italy,[19] which has complex DoFs as well. When we consider three translational parallel manipulators, Tsai's parallel manipulator[11] is worth mentioning. Although Tsai's manipulator has translations identical with that of DELTA, this manipulator is not the version of DELTA. Tsai's manipulator is the first design to solve the problem of UU chain. There is also another 3 translational DoFs parallel manipulator, Star, which is designed by Hervé based on group theory.[10] Such a kind of parallel manipulator has wide applications in industrial world, e.g. pick-and-place application, parallel kinematics machines, and medical devices.[12] In the family of spatial 3-DoF parallel manipulators, the manipulators with three rotational DoFs received much more attention.[20–23] They have been applied to the fields of camera-orienting and haptic devices.[24,25] Another type of 3-DoF parallel manipulator is that the moving platform is connected to the base through four legs. For such a mechanism, it usually consists of 3 identical actuated legs with 6 DoFs and one passive leg with 3 DoFs connecting the platform and the base, i.e. the degree of freedom of the mechanism is dependent on the passive leg's degree of freedom. One can improve the rigidity of this type of mechanism through optimization of the link rigidities to reach a maximal global stiffness and precision.[26] This type of mechanism has been applied to the design of a hybrid machine tool.[27] There are few 3-DoF fully parallel manipulators that can combine the spatial rotational and translational DoFs and be further with definite motion (being opposite to the complex motion) except for that of ***HALF*** with three non-identical legs.[8] All these spatial

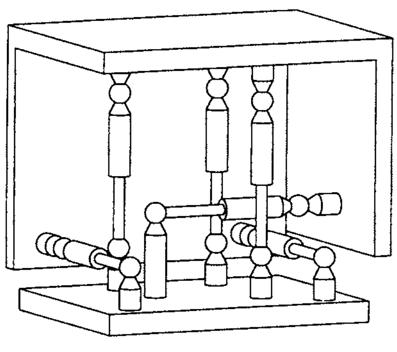

Fig. 2. The manipulator with 3-2-1 arrangement.

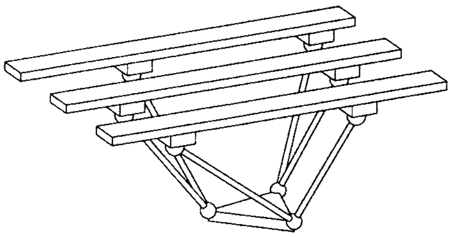

Fig. 3. The Hexaglide manipulator.

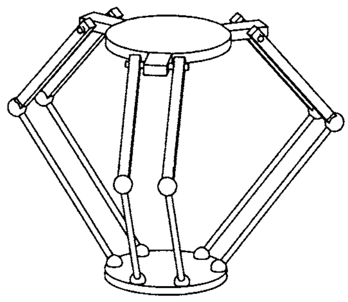

Fig. 4. The Pierrot's manipulator.

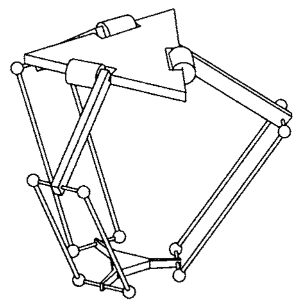

Fig. 5. The DELTA robot.

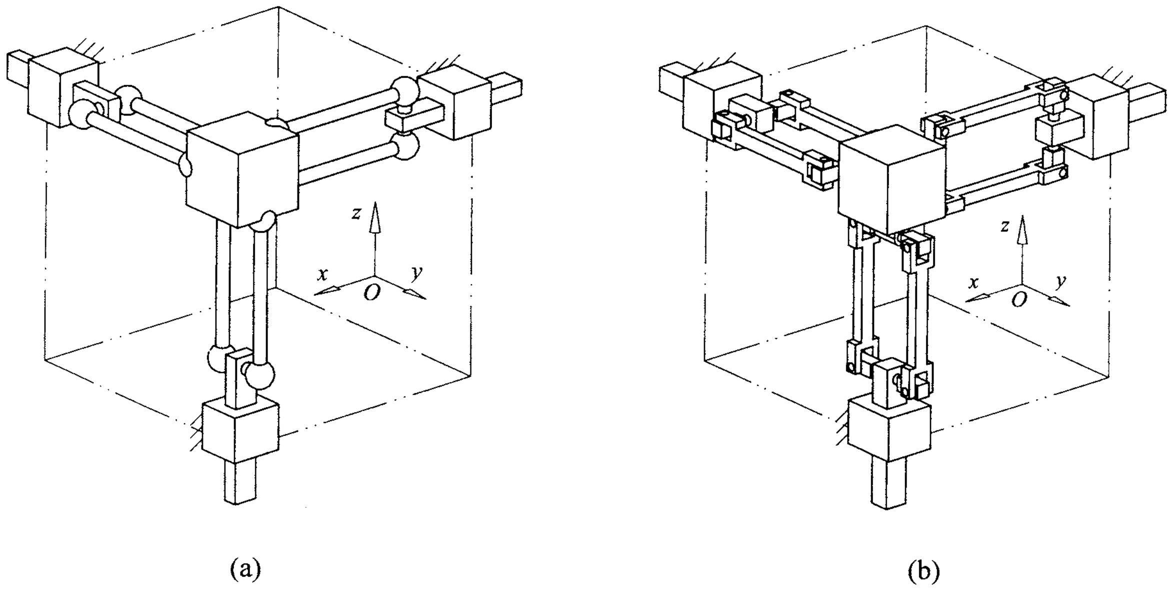

Fig. 6. The three translational DoFs parallel cube-manipulators.

parallel mechanisms have some disadvantages in common, i.e. singularity existing in the workspace, more complex forward kinematics problem, and no compliance center, which are very important for some applications. This paper attempts to present a three translational DoFs parallel manipulator which avoids these drawbacks.

The supposed design conceives for the above-mentioned manipulators can not only make us to understand the manipulators easily, but provide us new conceive to design a manipulator as well. For example, the 6-DoF parallel manipulators described in references [28,29] can be taken as the topology architectures of the manipulator shown in Fig. 1 by rearranging the six legs. This inspires us to design new parallel manipulators by rearranging the three legs of DELTA[9] and Tsai's parallel manipulator,[11] which are shown in Fig. 6. In Fig. 6(a), the moving platform is connected to the base by three legs with PRP^S chains, where P^S stands for the spatial four-bar parallelogram with four spherical joints, P prismatic joint and R revolute joint. In the design of Fig. 6(b), the three legs are with PRP^RR chains, where P^R denotes the planar four-bar parallelogram with four revolute joints. In those two designs, the three actuators are arranged according to the Cartesian coordinate system, which means that the actuating directions are normal to each other. And the joints connecting to the moving platform are located in three sides of a cube, for such reason we call this type of manipulator the parallel cube-manipulator. The manipulators also have three translational DoFs as in the cases of DELTA robot and Tsai's manipulator. But, they have some important characteristics being different from that of DELTA robot and Tsai's manipulator, which makes the new designs novel.

3. KINEMATICS PROBLEMS

A kinematics model of such parallel cube-manipulators can be developed as shown in Fig. 7. The center of the link in each of the three legs connected to the base by prismatic joints is denoted as B_i ($i=1, 2, 3$). And the center of the interval between the two spherical joints connecting the moving platform for each chain is denoted as P_i ($i=1, 2, 3$), which is in one of three sides of the cubic moving platform, respectively. B_i and P_i are the centers of the revolute joints in each leg for the design of Fig. 6(b). A fixed global reference system $\Re: o-xyz$ is located at the intersection point of three axes of actuated prismatic joints with the z-axis and the y-axis directed along the first and third actuation directions, respectively, as shown in Fig. 7. Another reference system $\Re: o'-x'y'z'$ is located at the center of the cubic moving platform. The z'-axis and y'-axis directed along P_1o' and P_3o', respectively, as shown in Fig. 7. Related geometric parameters are $o'P_i=r$ and $B_iP_i=L$, where $i=1, 2, 3$.

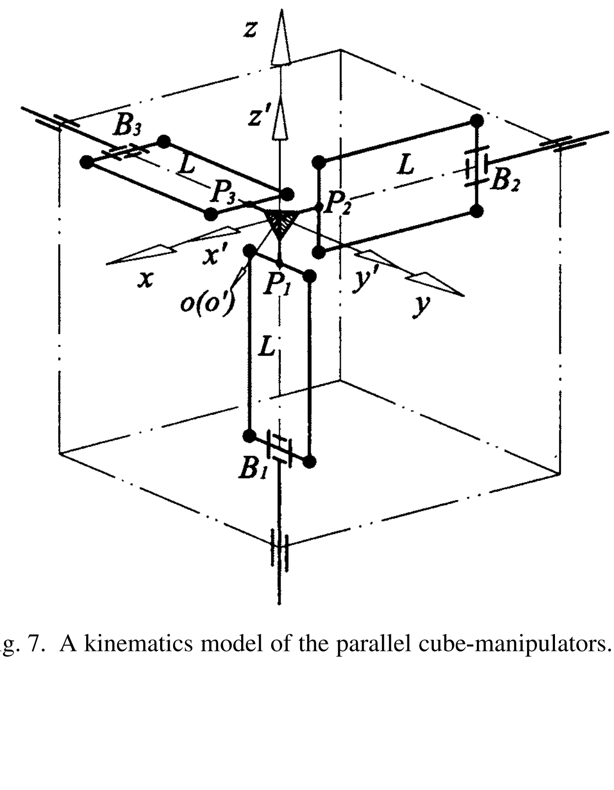

Fig. 7. A kinematics model of the parallel cube-manipulators.

3.1. Inverse kinematics problem

The objective of the inverse kinematics is to find the inputs of the manipulator with the given position of reference point o'. The position vector $\boldsymbol{c}_{\Re}$ of point o' in frame $\Re$ can be written as

$$\boldsymbol{c}_{\Re}=[x \quad y \quad z]^{\mathrm{T}} \tag{1}$$

As shown in Fig. 7, The coordinate of the point P_i in the frame $\Re'$ can be described by the vector $P_{i\Re'}$ $(i=1, 2, 3)$, which can be expressed as

$$\boldsymbol{p}_{1\Re'}=\begin{bmatrix}0\\0\\-r\end{bmatrix},\quad \boldsymbol{p}_{2\Re'}=\begin{bmatrix}-r\\0\\0\end{bmatrix},\quad \boldsymbol{p}_{3\Re'}=\begin{bmatrix}0\\-r\\0\end{bmatrix} \tag{2}$$

Then vectors $\boldsymbol{p}_{i\Re}$ $(i=1, 2, 3)$ in frame $O-xyz$ can be written as

$$\boldsymbol{p}_{i\Re}=\boldsymbol{p}_{i\Re'}+\boldsymbol{c}_{\Re}, \tag{3}$$

As shown in Fig. 7, vector $\boldsymbol{b}_{i\Re}$ $(i=1, 2, 3)$ is defined as the position vector of point B_i in frame $\Re$, and

$$\boldsymbol{b}_{1\Re}=\begin{bmatrix}0\\0\\z_1\end{bmatrix},\quad \boldsymbol{b}_{2\Re}=\begin{bmatrix}x_2\\0\\0\end{bmatrix},\quad \boldsymbol{b}_{3\Re}=\begin{bmatrix}0\\y_3\\0\end{bmatrix} \tag{4}$$

where z_1, x_2 and y_3 are the inputs of the manipulator, which will be denoted as ρ_1, ρ_2 and ρ_3 respectively. Then the inverse kinematics of the parallel cube-manipulator can be solved by writing following constraint equation

$$\|\boldsymbol{p}_{i\Re}-\boldsymbol{b}_{i\Re}\|=L, \quad i=1, 2, 3 \tag{5}$$

that is

$$x^2+y^2+(z-\rho_1-r)^2=L^2 \tag{6}$$

$$(x-\rho_2-r)^2+y^2+z^2=L^2 \tag{7}$$

$$x^2+(y-\rho_3-r)^2+z^2=L^2 \tag{8}$$

from which, if the position of the moving platform is specified, the inputs of the manipulator can be obtained as

$$\rho_1=\pm\sqrt{L^2-x^2-y^2}+z-r \tag{9}$$

$$\rho_2=\pm\sqrt{L^2-z^2-y^2}+x-r \tag{10}$$

$$\rho_3=\pm\sqrt{L^2-x^2-z^2}+y-r \tag{11}$$

from which one can see that there are eight inverse kinematics solutions for the manipulator. To get the inverse configuration as shown in Fig. 7, the sign "±" in each of the Eqs. (9)–(11) should be "–". Actually, any other assembly modes except for the one as shown in Fig. 7 will undoubtedly result in the interference between the moving platform and the legs. Therefore, the cube-manipulator can only be assembled as the configuration shown in Fig. 7, which means that there is only one inverse kinematics solution for the manipulator.

3.2. Forward kinematics problem

From Eqs. (6)–(8), we can see that, in each equation, the maximum degree of the polynomial is two, the coeffcient of which is 1, and, what is more, there is only one variable among x, y and z is with one degree. This allows us to express the two of three variables x, y and z with the third one very easily, e.g. x and y can be described as the function of z, that is

$$x=\frac{\rho_1+r}{\rho_2+r}z-\frac{(\rho_1+r)^2}{2(\rho_2+r)}+\frac{\rho_2+r}{2} \tag{12}$$

$$y=\frac{\rho_1+r}{\rho_3+r}z-\frac{(\rho_1+r)^2}{2(\rho_3+r)}+\frac{\rho_3+r}{2} \tag{13}$$

Substituting Eqs. (12) and (13) to Eq. (6) and rearranging it will lead to

$$az^2+bz+c=0 \tag{14}$$

Then there is

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{15}$$

in which

$$a=1+\left(\frac{\rho_1+r}{\rho_2+r}\right)^2+\left(\frac{\rho_1+r}{\rho_3+r}\right)^2$$

$$b=-(\rho_1+r)^3\left[\frac{1}{(\rho_2+r)^2}+\frac{1}{(\rho_3+r)^2}\right]$$

$$c=\frac{(\rho_2+r)^2}{4}+\frac{(\rho_3+r)^2}{4}+\frac{(\rho_1+r)^4}{4}\left[\frac{1}{(\rho_2+r)^2}+\frac{1}{(\rho_3+r)^2}\right]-L^2$$

x and y will be reached substituting Eq. (15) to Eqs. (12) and (13), respectively, if inputs ρ_1, ρ_2 and ρ_3 are given. And from Eqs. (12), (13) and (15) one can see that there are only two forward kinematics solutions for the manipulator. To obtain the foward configuration as shown in Fig. 7, the sign "±" in the Eq. (15) should be "+". The second forward kinematics solution, in which the sign "±" in the Eq. (15) is "–", will lead to the interference between the moving platform and legs. Comparing the forward kinematics problem with the result of a DELTA robot with linear actuators described in references [30,31], it is not difficult to find that the problem of the cube-manipulator studied in this paper is relatively more simple.

4. VELOCITY EQUATIONS AND JACOBIAN MATRIX

Equations (6)–(8) can be differentiated with respect to time to obtain the velocity equations, which leads to

$$(x-\rho_2-r)\dot{x}+y\dot{y}+z\dot{z}=(x-\rho_2-r)\dot{\rho}_2$$

$$x\dot{x}+(y-\rho_3-r)\dot{y}+z\dot{z}=(y-\rho_3-r)\dot{\rho}_3$$

$$x\dot{x}+y\dot{y}+(z-\rho_1-r)\dot{z}=(z-\rho_1-r)\dot{\rho}_1$$

Rearranging above three equations leads to an equation of the form

$$\boldsymbol{A}\dot{\boldsymbol{\rho}}=\boldsymbol{B}\dot{\boldsymbol{p}} \tag{16}$$

where $\dot{\boldsymbol{p}}$ is the vector of output velocities defined as

$$\dot{\boldsymbol{p}}=[\dot{x} \quad \dot{y} \quad \dot{z}]^{\mathrm{T}} \tag{17}$$

and $\dot{\rho}$ is the vector of input velocities defined as

$$\dot{\boldsymbol{\rho}}=[\dot{\rho}_2 \quad \dot{\rho}_3 \quad \dot{\rho}_1]^{\mathrm{T}} \tag{18}$$

Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are, respectively, the 3×3 inverse and forward Jacobian matrices of the manipulator and can be expressed as

$$\boldsymbol{A}=diag(a_{11}, a_{22}, a_{23}) \tag{19}$$

$$\boldsymbol{B}=\begin{bmatrix} a_{11} & y & z \\ x & a_{22} & z \\ x & y & a_{33} \end{bmatrix} \tag{20}$$

with $a_{11}=x-\rho_2-r$, $a_{22}=y-\rho_3-r$ and $a_{33}=z-\rho_1-r$. And if the matrix $\boldsymbol{A}$ is nonsingular, Eq. (16) can be rewritten as

$$\dot{\boldsymbol{\rho}}=\boldsymbol{J}\dot{\boldsymbol{p}} \tag{21}$$

where $\boldsymbol{J}$ is the Jacobian matrix of the manipulator,

$$\boldsymbol{J}=\boldsymbol{A}^{-1}\boldsymbol{B}=\begin{bmatrix} 1 & y/a_{11} & z/a_{11} \\ x/a_{22} & 1 & z/a_{22} \\ x/a_{33} & y/a_{33} & 1 \end{bmatrix} \tag{22}$$

5. SINGULARITIES

Singularities are particular configurations where the manipulator becomes uncontrollable. As it is well known, there are three kinds of singularities in multi-loop kinematics mechanisms,[32,33] which are determined by the analysis of inverse and forward Jacobian matrices $\boldsymbol{A}$ and $\boldsymbol{B}$:

(i) The first kind of singularity occurs when $\boldsymbol{A}$ becomes singular but $\boldsymbol{B}$ is invertible, i.e.

$$\det(\boldsymbol{A})=0 \quad \text{and} \quad \det(\boldsymbol{B})\neq 0 \tag{23}$$

This kind of singularity corresponds to the configuration in which the chain reaches either a boundary of its workspace or an internal boundary limiting different subregions of the workspace where the number of branches is not the same.

(ii) The second kind of singularity, occurring only in closed kinematics chains, arises when $\boldsymbol{B}$ becomes singular but $\boldsymbol{A}$ is invertible, i.e. when

$$\det(\boldsymbol{A})\neq 0 \quad \text{and } \det(\boldsymbol{B})=0 \tag{24}$$

which corresponds to the singularity being located inside the workspace of the manipulator. In such configuration, the output link is locally movable even when all the actuated joints are locked, and the output link cannot resist one or more forces or moments even when all actuators are locked.

(iii) The third kind of singularity occurs when both $\boldsymbol{A}$ and $\boldsymbol{B}$ become simultaneously singular. This singularity is of a slightly difference from the first two since it is not only configuration- but architecture-dependent as well.[32] This corresponds to configurations in which the chain can undergo finite motions when its actuators are locked.

From Eqs. (19) and (20), one can see that the first kind of singularity will occur if any one of the three points B_i is located in the plane deterrnined by the side of the cubic moving platform where P_i is located. In this condition, the parallelogram will also reach its singular configuration. But the manipulator cannot reach these configurations owing to mechanical limitations, which means that there is no such singularity within the workspace of a parallel cube-manipulator. It is more important that the second kind of singularity will not occur in the case of the parallel cube-manipulator and only $L=0$ will lead to the third kind of singularity. All in all, the parallel cube-manipulator has no singularities practically.

6. WORKSPACE ANALYSIS

Workspace is an important index to evaluate the working performance of a manipulator. How to determine the workspace is one of the most important issues in the design of a manipulator, especially for a parallel manipulator which is well known to have relatively small useful workspace. Therefore, the geometric determination of the workspace for a parallel manipulator attracts more and more researcher's attention.[34–36] The workspace of translational parallel manipulators has been studied by many researchers.[30,37–39] In general, it can always be determined geometrically for such manipulators, which is also same to the parallel cube-manipulators considered in this paper.

From Eqs. (6)–(8), one can see that if the inputs of the cube-manipulator are given, workspaces of the three legs are three spheres, which are centered at points $(0, 0, \rho_1+r)$, $(\rho_2+r, 0, 0)$ and $(0, \rho_3+r, 0)$, respectively. The radius of each of the three spheres is L. If the inputs ρ_i are specified as

$$\rho_i\in[\rho_{\min}, \rho_{\max}] \tag{25}$$

the workspace of each of the three legs of the cube-manipulator is then the enveloping solid of a sphere whose center is moving along a line between two points C_i and D_i, where $C_i(0, 0, \rho_{\min}+r)$ and $D_1(0, 0, \rho_{\max}+r)$ are for the first leg, $C_2(\rho_{\min}+r, 0, 0)$ and $D_2(\rho_{\max}+r, 0, 0)$ for the second leg, and $C_3(0, \rho_{\min}+r, 0)$ and $D_3(0, \rho_{\max}+r, 0)$ the third leg. And the workspace of the cube-manipulator is, then, the intersection of the three enveloping solids.

For example, the workspace of a cube-manipulator with the geometric parameters $r=260$, $L=1000$, $\rho_{\max}=-774$ and $\rho_{\min}=-1746$ ($|\rho_{\max}-\rho_{\min}|=972$) is shown in Fig. 8, the volume of which is 940022941.13, which can be obtained easily on the AutoCAD R14 platform.

7. STIFFNESS MATRIX AND COMPLIANCE STUDY

By virtue of what is called the *duality of kinematic and static*,[40] the forces and moments applied at the moving platform under static conditions are related to the forces or moments required at the actuators to maintain the equilib-

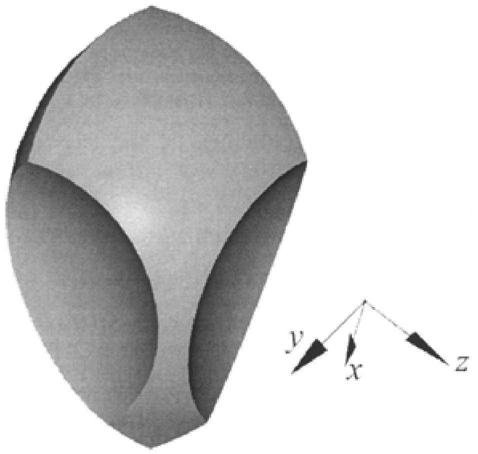

Fig. 8. The workspace of a three translational DoFs parallel cube-manipulator.

rium by the transpose of the Jacobian matrix J. We can write

$$\tau = \boldsymbol{J}^{\mathrm{T}} \boldsymbol{f} \tag{26}$$

where $\boldsymbol{f}$ is the vector of actuator forces or torques, and τ is the generalized vector of Cartesian forces and torques at the moving platform.

In the joint coordinate space, a diagonal stiffness matrix $\boldsymbol{K}_p$ is defined to express the relationship between the actuator force or torques $\boldsymbol{f}$ and the joint displacement vector $\Delta\boldsymbol{q}$ according to

$$\boldsymbol{f} = \boldsymbol{K}_p \, \Delta\boldsymbol{q} \tag{27}$$

with

$$\boldsymbol{K}_p = \begin{bmatrix} k_{p1} & 0 & 0 \\ 0 & k_{p2} & 0 \\ 0 & 0 & k_{p3} \end{bmatrix} \tag{28}$$

in which k_{pi} is a scalar representing the stiffness of each of the actuators.

In the operational coordinate space, we define a stiffness matrix $\boldsymbol{K}$ which relates the external force vector τ to the output displacement vector $\boldsymbol{D}$ of the moving platform according to

$$\tau = \boldsymbol{K}\boldsymbol{D} \tag{29}$$

The Eq. (21) can also describe the relationship between the joint displacement vector $\Delta\boldsymbol{q}$ and output displacement vector $\boldsymbol{D}$, i.e.

$$\Delta\boldsymbol{q} = \boldsymbol{J}\boldsymbol{D} \tag{30}$$

From Eqs. (26), (27) and (30), we get

$$\tau = \boldsymbol{J}^{\mathrm{T}} \boldsymbol{K}_p \boldsymbol{J} \boldsymbol{D} \tag{31}$$

Thus, the stiffness matrix $\boldsymbol{K}$ is expressed as

$$\boldsymbol{K} = \boldsymbol{J}^{\mathrm{T}} \boldsymbol{K}_p \boldsymbol{J} \tag{32}$$

Compliance is defined as the ability of a manipulator to react to external forces. This characteristic is very important for a mechanism intended to be as a remote center compliance (RCC) device or perform assembly tasks.[28,41] For a mechanism to have a compliance characteristic, it must have configurations where the stiffness matrix $\boldsymbol{K}$ should be a diagonal matrix. Physically, at the compliance point, the deformation of the device due to external force or torque should occur only along the direction of applied external force or torque.

For the parallel cube-manipulator considered in this paper, in the particular position where $x=y=z=0$, Jacobian matrix $\boldsymbol{J}$ becomes an identity matrix, which leads $\boldsymbol{K}$ to be

$$\boldsymbol{K} = \boldsymbol{K}_p = \begin{bmatrix} k_{p1} & 0 & 0 \\ 0 & k_{p2} & 0 \\ 0 & 0 & k_{p3} \end{bmatrix}$$

which is a diagonal matrix. Then, in position $x=y=z=0$, the manipulator has the compliance characteristic. Particularly, when $k_{p1}=k_{p2}=k_{p3}$, the parallel manipulator behaves like a both velocity and stiffness isotropic setup.

8. APPLICATION EXAMPLE

From above analysis, one can reach the results that the inverse and forward kinematics problems can be described in closed forms and can be solved more easily, and, especially, each of the solutions of which is unique. Practically, there is no singularity in the workspace. Moreover, one of the most important results is that there exists a compliance center in the workspace, which is the position where $x=y=z=0$. At this point, the parallel cube-manipulator behaves like a both velocity and stiffness isotropic setup.

These characteristics make the parallel cube-manipulator be well adapted to the applications of micro-motion manipulators[18,42] remote center compliance (RCC) devices,[41] precision assembly machines,[28] parallel kinematics machines,[43] and so on. To be applied to the parallel kinematics machine, the system's stiffness is one of the most important problems. The advantage of the presented parallel manipulators is that the stiffness can be improved by increasing the redundant constraints, e.g. the architectures as shown in Fig. 9. In Fig. 9(a), the design is the revised version of that in Fig. 6(a), where there are three spherical-bar-spherical chains instead of two such chains in each leg. In Fig. 9(b), the parallelogram in Fig. 6(b) is divided into two or more parallelograms for each leg. The kinematics model of each of the mechanisms is identical with that of each of manipulators shown in Fig. 6. What is more, the revision has no any negative influence on the kinematics, workspace, and other performances of the manipulator. Needless to say, the more the redundant constraint the leg has, the higher the stiffness the manipulator will get, and, in relative terms, the higher the fabricate accuracy it needs. The manipulator mechanisms can be designed as three-axis parallel kinematics machines or five-axis hybrid machines with a two-axis wrist joint attached to the moving platform. Because the mechanisms have more simple forward kinematics problem with closed forms, the calibration of the machines based on forward kinematics will be more easy.

As another example, Fig. 10 shows the application in micro-motion manipulators, in which all joints are replaced by flexure joints, which are transmission elements free

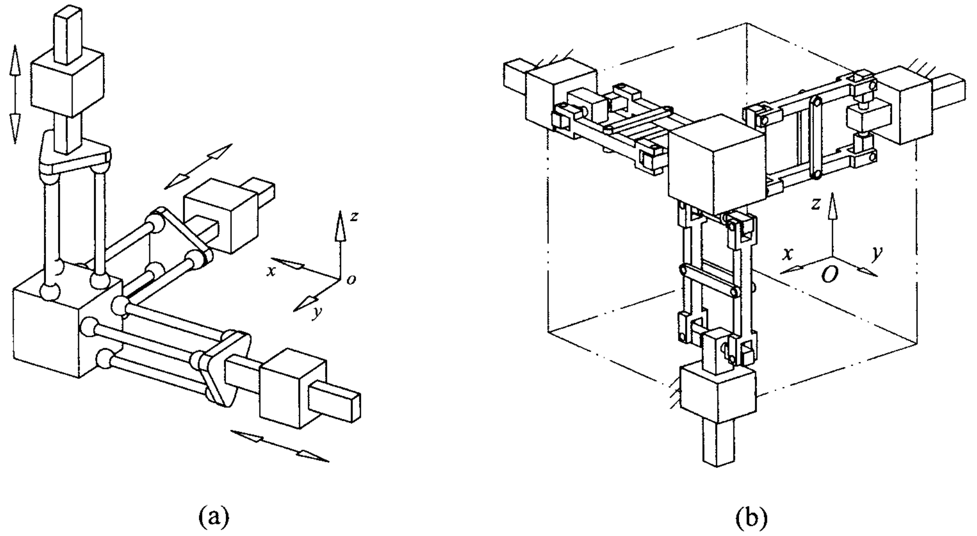

Fig. 9. The parallel cube-manipulators with redundant constraints.

without backlash, friction and hysteresis,[44] and the actuators are replaced by piezo actuators. Because the micro-motion manipulator is based on the architecture of the parallel manipulator, the analysis method of a parallel manipulator can be directly advanced to that of a micro-motion manipulator, as it presented in reference [18]. The design theory is also adapted to such kind of device, e.g. the method proposed by Liu[42] to the design of a six-DoF micro-motion manipulator. Generally, the design of such devices is usually a process to determine the geometric parameters based on some performance criteria, such as workspace, dexterity and stiffness. From above analysis, one can see that the involved geometric parameters of the manipulator are r and L, which, as well known, will have close influence on the workspace and the performance of the manipulator. A micro-motion manipulator usually has very small workspace, for which one can select one point, usually the original point with $x=y=z=0$, to evaluate the performance, e.g. the conditioning index, of such device.[42] The conditioning index μ is defined as the reciprocal of the condition number of Jacobian matrix $\boldsymbol{J}$, that is

$$\mu=1/\kappa \tag{33}$$

where κ is the condition number of the Jacobian matrix, and $\kappa=\|\boldsymbol{J}^{-1}\|\|\boldsymbol{J}\|$, in which $\|\cdot\|$ denotes the any norm of a matrix. The index μ is an all-around index that it can evaluate not only the dexterity, isotropy, but the static stiffness of the mechanism as well.[21,38] Substituting $x=y=z=0$ to Eq. (22) leads to

$$\boldsymbol{J}_0=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

which results in $\mu=1$. This means that the micro-motion manipulator is characterized by stiffness and velocity isotropy. What is more, this gives us an important information that, to design such kind of micro-motion manipulator, only the workspace can be considered, geometric parameters r and L have no influence on other kinematics performances, which mostly simplifies the design process of the micro-motion manipulator. As it shows in Section 6, to determine r and L with respect to a desired workspace is very easy, because the workspace can be obtained geometrically. Therefore, for the application in the field of the micro-motion manipulator, the device has the

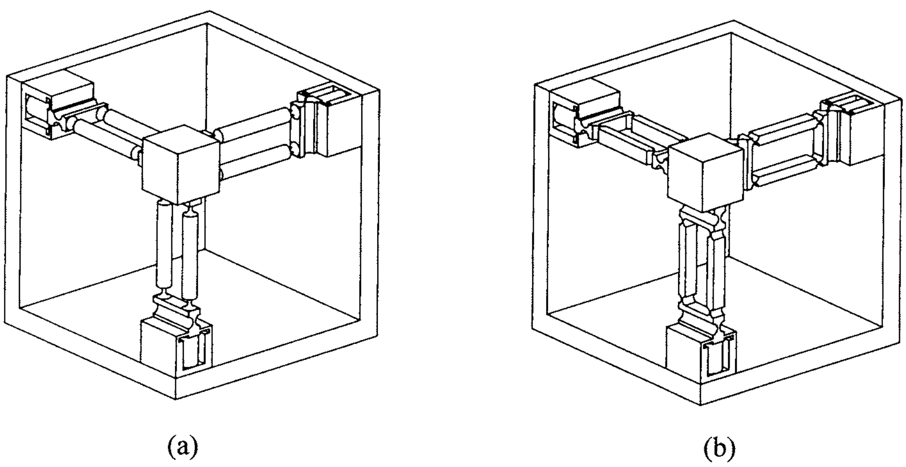

Fig. 10. Micro-motion manipulators based on the parallel cube-manipulators.

advantages of compliance, stiffness and velocity isotropy, and simple design process.

9. CONCLUSIONS

In this paper, a kind of three translational DoFs parallel cube-manipulator is presented. The manipulators, with normal actuation setups, are the topologies of DELTA robot and Tsai's manipulator, respectively. The kinematics problems, singularity, workspace, compliance characteristic of the manipulator are investigated here. The kinematics analysis of the manipulator shows that the solution for the inverse and forward kinematics is unique for the reason of avoiding interference. The manipulators have the advantages of high compactness and stiffness, no singularities in the workspace, relatively more simple forward kinematics, and existence of a compliance center $x = y = z = 0$. And the parallel cube-manipulator can be applied to the fields of micro-motion manipulators, remote center compliance (RCC) devices, assembly, parallel kinematics machines, and so on. Other advantages of this type of parallel manipulator will be investigated in the future work.

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References

1. D. Stewart, "A platform with six degrees of freedom", *Proc Inst Mech Eng* **180**, 371–386 (1965).
2. F. Behi, "Kinematics analysis for a six-degree-of-freedom 3-PRPS parallel manipulator", *IEEE J. of Robotics and Automation* **4**, No. 5, 561–565 (1988).
3. J. Hudgens and D. Tesar, "A fully-parallel six degree-of-freedom micromanipulator: kinematics analysis and dynamic model", *Proc. 20th Biennial ASME Mechanisms Conf.* (1988) **Vol. 15–3**, pp. 29–38.
4. A. Romiti and M. Sorli, "A parallel 6-DoF manipulator for cooperative work between robots in deburring", *Proc. of the 23rd Int. Symp. On Indusrial Robots*, Barcelona, Spain (1992) pp. 437–442.
5. Austad, "Arm Device", *IPN No WO 87,03239* (June 4, 1987).
6. F. Pierrot and O. Company, "H4: a new fahnily of 4-dof parallel robots", *Proc. of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Atlanta, USA (1999) pp. 508–513.
7. J. Kim *et al.*, "Eclipse-II: a new parallel mechanism enabling continuous 360-degree spinning plus three-axis translational motions", *IEEE Transactions on Robotics and Automation* **18**, No. 3, 367-373 (2002).
8. Xin-Jun Liu *et al.*, On the analysis of a new spatial three degrees of freedom parallel manipulator, *IEEE Transactions on Robotics and Automation* **17**, No. 6, 959–968 (2001).
9. R. Clavel, "DELTA: a fast robot with paralliel geometry", *18th Int. Symp. on Industrial Robots* (1988) pp. 91–100.
10. J. M. Hervé, "Group mathematics and parallel link mechanisms", *IMACS/SICE Int. Symp. On Robotics, Mechatronics, and Manufacturing Systems* (1992) pp. 459–464.
11. L. W. Tsai and R. Stamper, "A parallel manipulator with only translational degrees of freedom", *ASME 1996 Design Engineering Technical Conference*, Irvine, CA (1996) **96-DETC-MECH–1152**.
12. Ilian Bonev, "The Delta parallel robot-the story of success", http://www.parallelmic.org/Reviews/ ReviewO02p.html (2001).
13. E. Pernette and R. Clavel, "Parallel robots and microrobotics", *ISRAM'96*, Montpellier (1996) pp. 535–542.
14. M. Honegger, R. Brega and G. Schweitzer, "Application of a nonlinear adaptive controller to a 6 dof parallel manipulator", *Proceedings of IEEE International Conference on Robotics & Automation*, San Francisco, CA (2000) pp. 1930–1935.
15. F. Pierrot, P. Dauchez and A. Fournier, "Towards a fully-parallel 6 d.o.f. robot for high speed applications", *Proceedings of IEEE International Conference on Robotics & Automation*, Sacramento, California (1991) pp. 1288–1293.
16. K. H. Hunt, "Structural kinematics of in-parallel-actuated robotarms", *Trans. ASME, J: Mech., Transmis., Automa. In Des.* **105**, 705–712 (1983).
17. Z. Huang, J. Wang and Y. F. Fang, "Analysis of instantaneous motions of deficient-rank 3-RPS parallel manipulators", *Mechanism and Machine Theory* **37**, 229–240 (2002).
18. Y. Ohya *et al.*, "Development ot 3-DOF finger module for micro manipulation", *Proceedings of the 1999 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Kyongju, Korea (1999) pp. 894–899.
19. M. Ceccarelli, "A new 3 D.O.F. spatial parallel mechanism", *Mechanism and Machine Theory* **32**, No. 8, 895–902 (1997).
20. C. Gosselin and J.Angeles, "The optimum kinematic design of a spherical three-degree-of-freedom parallel manipulator", *J. Mech. Transm. Autom. Des.* **111**, No. 2, 202–207 (1989).
21. Xin-Jun Liu, Z.-L. Jin and F. Gao, "Optimum design of 3-DOF spherical parallel manipulators with respect to the conditioning and stiffness indices", *Mechanism and Machine Theory* **35**, No. 9, 1257–1267 (2000).
22. R. D. Gregorio, "A new family of spherical parallel manipulators", *Robotica* **20**, No. 4, 353–358 (2002).
23. X. Kong and C.M. Gosselin, "Type synthesis of 3-DOF spherical parallel manipulators based on screw theory", *Proceedings of DET'02 ASME 2002 Design Engineering Technical Conferences and Computer and Information in Engineering Conference*, Montreal, Canada (2002) **DETC2002/MECH–34259**.
24. C. M. Gosselin and St-Pierre, "Development and experimentation of a fast 3-DOF camera-orientating device", *Int. J. Robotics Research* **16**, 619–630 (1997).
25. L. Birgler *et al.*, "ShaDe, a new 3-DOF haptic device", *IEEE Transactions on Robotics and Automation* **18**, No. 2, 166–175 (2002).
26. D. Zhang and C.M. Gosselin, "Kinetostatic modeling of parallel mechanisms with a passive constraining leg and revolute actuators", *Mechanism and Machine Theory* **37**, 599–617 (2002).
27. H. K. Tonshoff, H. Grendel and R. Kaak, "Structure and characteristics of the hybrid manipulator Georg V", **In**: (C. R. Boer, L. Molinari-Tosatti and K. S. Smith editors) *Parallel Kinematic Machines* (Springer-Verlag, London Limited, 1999) pp. 365–376.
28. E.-M. Dafaoui *et al.*, "Analysis and design of a six-DoF parallel manipulator, modeling, singular configurations, and workspace", *IEEE Transactions Robotics and Automation* **14**, No. 1, 78–92 (1998).
29. F. Gao, Z. Lin, Xin-Jun Liu *et al.*, "A six-DoF virtual-axis machine tool", *China Patent: CN 1261018A* (Nov. 1999).
30. Xin-Jun Liu, "Mechanical and kinematics design of parallel robotic mechanisms with less than six degrees of freedom",

Postdoctoral Research Report (Tsinghua University, Beijing, China, 2001).
31. F. Pierrol *et al.*, "DELTA: a simple and eff cient parallel robot", *Robotica* **8**, Part 1, 105–109 (1990).
32. C.M. Gosselin and J. Angeles, "Singularity analysis of closed loop kinematic chains", *IEEE Trans. on Robotics and Automation* **6**, No. 3, 281–290 (1990).
33. H. R. M. Daniali, P. J. Zsombor-Murray and J. Angeles, "Singularity analysis of a general class of planar parallel manipulators", *Proceedings of IEEE Iternational Conference on Robotics and Automation*, Nagoya, Japan (1995) pp. 1547–1552.
34. C. Gosselin, "Determination of the workspace of 6-dof parallel manipulators", *Trans. ASME J. Mech. Design* **112**, 331–336 (1990).
35. I. A. Bonev and J. Ryu, "A geometrical method for computing the constant-orientation workspace of 6-PRRS parallel manipulators", *Mech. Mach. Theory* **36**, No. 1, 1–13 (2001).
36. J.-P. Merlet, "Determination of 6D Workspace of Gough-Type Parallel Manipulator and Comparison between Different Geometries", *Robotics Research* **18**, No. 9, 902–916 (1999).
37. F. Sternheim, "Tridimensional computer simulation of a parallel robot, result for the 'DELTA4' machine", *Proc. Int. Symposium on Industrial Robots*, Lausanne, Switzerland (April 1988) pp. 333–340.
38. Xin-Jun Liu, "The relationships between the performance criteria and link lengths of the parallel manipulators and their design theory", *Ph.D Thesis* (Yanshan University, Qinhuangdao, China, 1999).
39. A. Tremblay and L. Baron, "Geometrical synthesis of star-like topology parallel manipulators with a genetic algorithm", *Proceedings of IEEE International Conference on Robotics and Automation*, Michigan (1999) pp. 2446–2451.
40. K. J. Waldron and K. H. Hunt, "Series-parallel dualities in actively coordinated mechanisms", *Robotics Research* **4**, 175–181 (1988).
41. Whee-ku Kim, Jun-Yong Lee and Byung Ju Yi, "Analysis for a planar 3 degree-of-freedom parallel mechanism with actively adjustable stiffness characteristics", *Proceedings of IEEE International Conference on Robotics and Automation*, New Mexico (1997) pp. 2663–2670.
42. Xin-Jun Liu *et al.*, On the design of 6-DoF parallel micromotion manipulators, *Proceedings of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Hawaii, USA (Oct. 2001) pp. 343–348.
43. T. Huang, X. Zhao and D.J. Whitehouse, "Stiffness estimation of a tripod-based parallel kinematic machine", *IEEE Transactions on Robotics and Automation* **18**, No. 1, 50–58 (2002).
44. E. Pernettez *et al.*, "Design of parallel robots in microrobotics", *Robotica* **15**, Part 3, 417–420 (1997).