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Robust control of vertical motions in ultra-high rise elevators ☆

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Abstract

The advent of ultra-high rise buildings has brought control over elevator vertical motion to the forefront. Unlike traditional low/mid-rise elevators, relatively high speed coupled with long rope lengths result in the need to address flexible low-frequency modes and non-linear dynamics. Research in this direction has only been initiated recently and primarily confined to the industry.

This paper presents a practical methodology for designing high-performance controllers for elevator vertical motion for high-rise buildings. The methodology is directed towards satisfying several needs including scalability and ease of tuning of the control system. The former is important for adaptability to different hoistways, while the latter becomes necessary on account of performance degradation, experienced due to normal wear and tear. This is accomplished by developing a scalable lumped parameter model at several ultra-high rise hoistways leading to an alternate scalable empirical model based on a few prominent features in the vertical dynamics. A tunable controller based only on these features is developed. Simulation studies show that the controller meets a set of standardized tests that are typically used for evaluating elevator performance. A problem that arises in an ultra-high rise on account of the changing nature of dynamics as the elevator transits from one floor to another leading to the question of closed-loop non-linear stability is not a feature of the standardized tests. The problem of stability around a trajectory is reduced to a multi-variable Popov criterion and the tunable controller is shown to meet these requirements

Keywords: Robust control; Tunable control; Stability analysis; Popov criterion; Vertical dynamics

1. Introduction

Ultra-high rise buildings are becoming increasingly common (see Cesar, Thornton, & Joseph, 1997; Fortune, 1992, 1995) with buildings such as the Petronas Towers in Malaysia (1480 ft) and the Shanghai World Financial Center in China (1510 ft). In the absence of any effective and safe alternative, the conventional roped-elevator technology will have to be used for high-rise buildings (see Roberts (1998), for a discussion on the design of effective transportation systems for tall buildings). The advent of ultra-high rises has spawned research and development in the area of modeling, identification, control and actuation for elevator-motion control in ultra-high rise buildings (see Li, Niemann, & Wang, 1998; Roberts, 1998; Venkatesh & Cho, 1998; Tanaka, Nishimura, & Wang, 1998; Nai, Forsythe, & Goodall, 1994, 1995; Matsukura & Watanabe, 1992, So & Chow, 1996). There are several factors that differentiate ultra-high rise elevator control from low-mid rises and this paper will focus only on vertical motion here.

In low-mid rises, rope dynamics do not play an important role in characterizing the vertical dynamics on account of two principal reasons. The relatively short rope length, coupled with modest vertical speed, helps in simplifying the vertical dynamics to a point that the dynamics between motor torque and car velocity predominantly behaves as an integrator (inertia element). This is because a shorter rope length and lower speed generally implies that the rope modes are at relatively high frequencies and are insignificant in a closed-loop control context. Rope dynamics plays a significant role in vertical motion of an ultra-high rise elevator beyond the fact that the rope modes occur at lower frequencies and need to be confronted. As the

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elevator transits from one floor to another, the "effective length" of the rope changes and with it, the vertical dynamics also changes; thus, the dynamics is inherently non-linear.

In view of these factors, in the traditional elevator literature (see Aldaia, Aranbru, & Pagalday, 1996; Strakosch, 1967; Barney, 1986; Teramoto, Nakamura, & Wantanabe, 1996 and references therein), vertical motion control is not considered as a significant problem. Indeed, the control system employs a SISO PID control loop (see Teramoto et al., 1996; Barney, 1986; Nai et al., 1994, 1995) and does not use a car position sensor, and often, control is not explicitly based on any underlying model of the system. The traditional elevator literature has been primarily concerned with systems engineering, cost and efficiency, noise and vibration, transportation, traffic and service issues, and accurate position and control of specific components such as drives, doors, etc. (see Strakosch, 1967; Barney, 1986; Barney & Santos, 1977). The advent of ultra-high rises has, for the first time, brought vertical dynamics and control into focus.

There have been limited attempts at modeling, identification and control design for ultra-high rise elevators and a brief survey of these is given in the sequel. At the center of the development of control techniques for high-speed elevator control for ultra-high rises, are the improvements in actuation and drive technologies along with the better microprocessors (see Watanabe, Terazono, & Suzuki, 1992; Suzuki, Iwata, & Yonemoto, 1996) which have helped in increasing the actuation bandwidth to a level that is necessary for control of vertical motion. Since ultra-high rise elevator systems are a fairly recent phenomena, the literature in the area of modeling of elevator vertical dynamics is new (see Chi & Shu, 1991; Roberts, 1998; So & Chow, 1996), and not all of these are motivated by control of vertical motion. One of the first attempts towards modeling vertical motion was made in Chi and Shu (1991). Herein, the vertical dynamics of a simple situation of an elevator car hanging from a rigid drive sheave is considered. However, these models are extremely complicated and do not appear tractable for control design. The first effort towards obtaining control-oriented models was made in Roberts (1998), Venkatesh and Cho (1998), Cho and Rajamani (2001) where a lumped system finite-element based technique was used and this paper will be mainly concerned with this model. The difficulty in dealing with this model is due to a large number of parameters that characterize the model arising from accounting for a large number of physical components that comprise an elevator system. These parameters need to be accurately known for high performance and validation, and control-oriented identification becomes an issue. Venkatesh and Cho (1998) have proposed a methodology to identify and validate such a model, upon which the controller proposed in this paper is based. In the area of control-system design, again, a limited number of attempts (see Li et al., 1998; Tanaka et al., 1998; Roberts, 1998; Venkatesh & Cho, 1998; Cho & Rajamani, 2001; Nai et al., 1994, 1995) have been made. However, some such as Nai et al. (1994, 1995), primarily focus on the issue of control of vibration in a high-speed elevator. The others deal with vertical motion but have severe limitations. First, they primarily deal with a linear model and stability for the non-linear model has not been analyzed. They are impractical because they involve solving large-scale optimization problems resulting in a control system of extremely high order. This does not allow quick adaptability to different hoistways and simple tuning of the control system from time to time, which becomes necessary on account of wear and tear.

The principal contribution of this paper is a comprehensive and practical methodology for vertical motion control in ultra-high rise elevators. The control design methodology is shown to meet the performance and stability requirements. The control system can be customized to any hoistway and is readily tunable to maintain optimal performance in the context of wear and tear experienced in any typical hoistway. These objectives are accomplished in three steps.

- A lumped-parameter model is developed at several ultra-high rise hoistways.
- A control system based on only some prominent features, which can be readily identified with simple experiments, is developed.
- The control system is shown to meet a set of standardized set of tests that are typically used to evaluate elevator vertical ride quality. The question of stability, on account of changing dynamics, involves non-linear closed-loop stability around any trajectory. It is shown that the stability problem can be reduced to well-known multi-variable Popov criterion, which is verified numerically.

2. Summary of modeling, identification and validation

Modeling, identification and validation of ultra-high rise elevators are described in detail in Venkatesh and Cho (1998), Cho and Rajamani (2001). In this section, only the topics pertinent to this paper are briefly summarized for completeness.

The basic lumped-parameter model for a roped elevator-dynamic system with its respective components, illustrated in Roberts (1998), Venkatesh and Cho (1998), Cho and Rajamani (2001), is adopted in this paper. It consists of four major inertial elements: the drive sheave, elevator car, counterweight, and compensation sheave. Each of the major inertial elements is connected together with rope segments whose lengths vary as the elevator car moves up and down the hoistway. A three DOF model of the elevator car is assumed to represent the elevator frame, cab, and rope hitch. The mass of the car can vary depending on the number of passengers. The structural dynamics of the hoistway ropes is modeled using a lumped-mass approximation to capture the finite-force transmission delays in each of the four different rope segments (each of which is a set of multiple ropes). Each rope segment is of a different type and therefore, has different parameter values for stiffness, mass and damping. Viscous friction dampers are accounted for, in parallel with each spring. In addition, stiffness and damping to account for cab-isolation pads, and damping to account for dampers connected to an inertial ground for the drive sheave (Cds), compensation sheave rotation (Ccs) and translation (Cmm), counterweight (Ccwt), and elevator cab (Ccar) are also included in the model. In general, the elevator hoistway dynamics can be represented as a parameter-varying linear state space model of the form

$$y_p| = G(\theta, l)\, u_p, \quad \theta \in \Theta,$$

where l is the instantaneous vertical position of the elevator, generally a time varying quantity. The dynamics at each location, l, of the car is linear but the dynamics, as such, is non-linear as the car transits from one location to another. The linear transfer function at each location is characterized as follows:

$$\dot{x}_p(t) = A_p(\theta, l)\, x_p(t) + B_p(\theta, l)\, u_p(t),$$
$$y_p(t) = C_p(\theta, l)\, x_p(t), \quad \theta \in \Theta. \tag{1}$$

The plant model has control input (u_p) given by the torque T to the drive sheave motor and an exogenous disturbance F in the form of payload.

The parameters of (1) are determined via the methodology proposed in Venkatesh and Cho (1998), Cho and Rajamani (2001), and the results show that the experimental transfer-function estimates compare extremely well at all the locations with the model estimates, as shown in Fig. 1.

With a validated model, it is now possible to attempt to define some of the features that are significant for control design. The idea is that these features can be determined directly instead of going through the process of identifying and validating parameters in the lumped parameters. In this way, a controller, designed primarily based on these features, can be readily customized and tuned to any hoistway. The most important set of features is the behavior of the first two rope modes (1 and 3 Hz modes at the bottom for the hoistway of Fig. 1), with the position of the elevator car. The first elevator-system mode is a rigid-body mode and it is necessary to know the DC gain only. In summary, the features that are significant from a control viewpoint are the DC gain, the first two rope modes and their behavior. These could be determined, for instance, by means of either a sine-sweep or random-input tests at different points in the hoistway. To get a feel for how the various modes behave, the behavior of the first rope mode is illustrated in Fig. 2 for the transfer function between the velocity of the drive sheave, V_{ds}, to the velocity of the car, V_{car}. The stiffness and damping are the absolute and real values of the poles at the corresponding frequency. A careful observation shows that the damping coefficient increases faster than the stiffness, as a function of the instantaneous location, l, of the elevator car. This leads one to believe that the system gets "more stable", as the elevator car approaches the top of the hoistway. Indeed, this notion will be formalized and it is shown that an LTI controller, based on the dynamics at the bottom of the hoistway, meets the requirements for non-linear closed-loop stability. Also, notice that the behavior can be approximated well by a first-order polynomial up to a height of 60 floors corresponding to a height of 200 m. Notice also from Fig. 2, that the stiffness and damping start to flatten out beyond this height. This should not come as a surprise as the rope begins to have less effect with the equalizer-spring dictating, accounting for more of the dynamics.

3. Control design

In this section, a tunable LTI control system is developed to meet the stability and performance requirements. The performance requirements will be based on simulations of some standard tests, such as re-leveling and ride quality, that are common in the elevator industry. Stability will have two aspects, one that deals with local asymptotic stability and the other, dealing with non-linear stability. The requirement for local stability arises on account of the need to meet some of the performance requirements in the face of parametric and non-parametric uncertainty. Parametric uncertainty arises because the parameter values for the stiffness and damping coefficients vary over time due to degradation and as mentioned earlier, this could be as large as 20%; non-parametric uncertainty arises because the linear dynamics at high frequencies is not accurately known. For instance, the vertical dynamics shown in Fig. 1 approximates the estimated transfer function up to a frequency of 12 Hz and beyond, in which the residual error increases substantially. The range of uncertainty in the parameters, θ, is based on empirical knowledge acquired by independently studying the behaviors of the system components over a long period of time. Non-linear stability arises in ultra-high rises, on account of the changing nature of the dynamics.

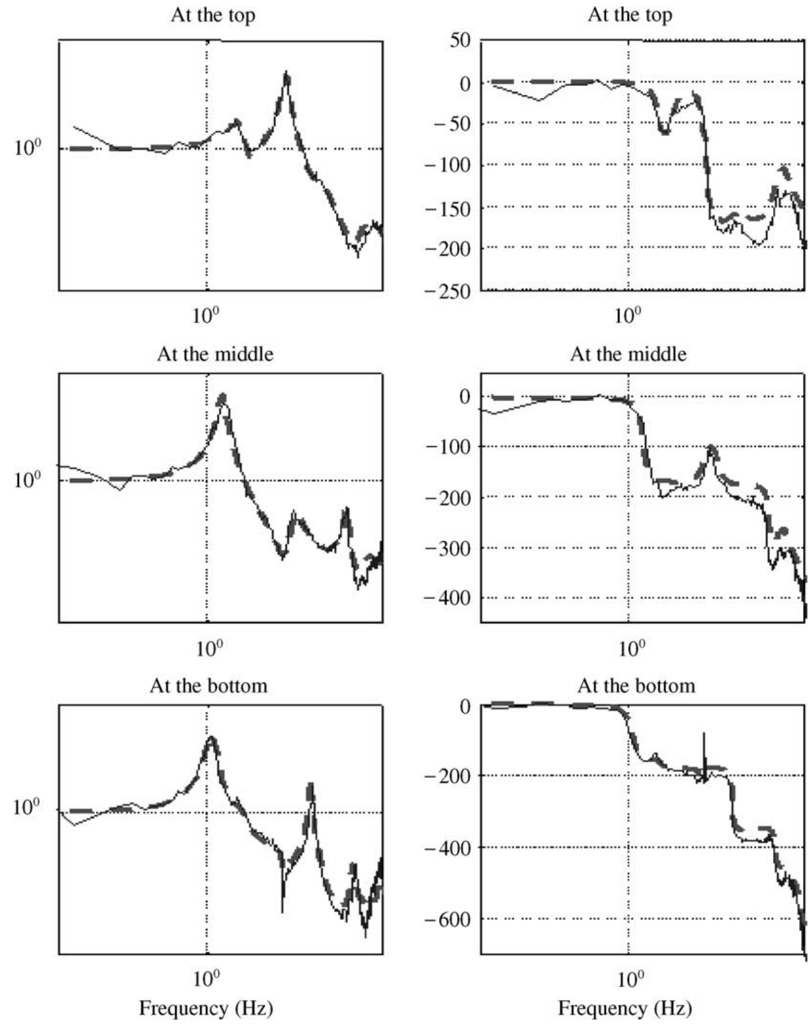

Fig. 1. Model-validation results with sinusoidal (dots) and random inputs (dashed) for the elevator system; comparison of physics-based model (solid) with experimental data; illustration of movement of modes with elevator position.

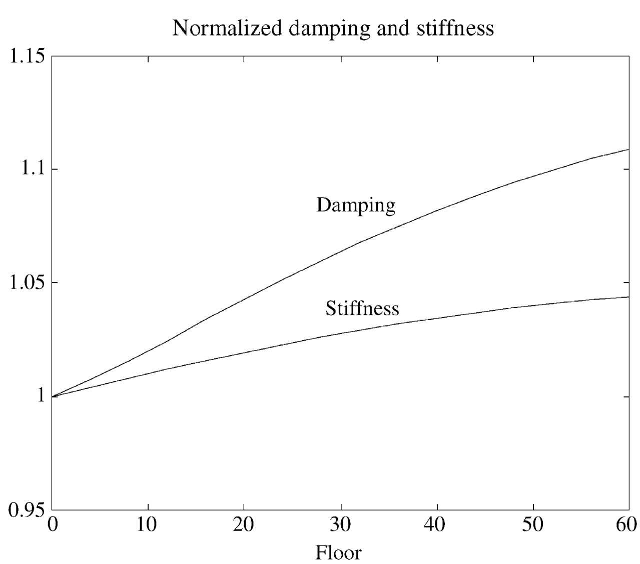

Fig. 2. Variation of the first rope mode observed in drive sheave→car velocity transfer function.

The elevator system from a control viewpoint provides two sensor measurements, car position and drive-sheave velocity, and a single control in the form of torque actuation, through a motor to the drive sheave. There is a disturbance in the form of a payload, F. To evaluate the performance for the elevator vertical ride, a set of standardized tests is often used in practice. Re-leveling error and vertical ride quality are of primary concern here. These will be described very briefly here and the reader is referred to Zubia (1996), Peters (1995), Beldiman, Wang, and Bushnelll (1998), Roberts (1998) for further details.

Re-leveling error is a measure of the elevator system response to increase or decrease of payload in the elevator car. It is required that the elevator car be regulated to within 6 mm of its target floor position quickly, in response to any payload ranging from 0% to 100% capacity. A smooth profile as shown in Fig. 3(a) is typically used as a basis for measuring re-leveling. This profile simulates the effect of increasing the payload

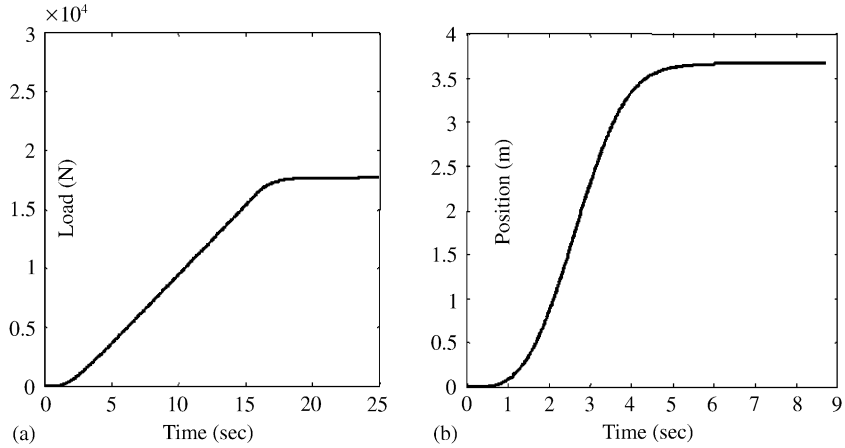

Fig. 3. (a) Typical load profile for simulating re-leveling performance; (b) a one-shuttle run profile used as a reference trajectory dictation for evaluating vertical ride quality.

from 0% to 50% in 15 s. This is about the time it takes 18 people to get into an elevator. These simulations need to be carried out at all locations of the elevator car. As the payload is increased, the elevator car sags below that floor level and it is required that the elevator control system quickly responds to the load and re-level the elevator car so that the car is never more than 6 mm away from the floor level.

Ride quality has been characterized from different viewpoints but this paper focuses on vertical motion only. It turns out that humans are extremely sensitive to sudden changes in acceleration (large jerks). For this reason, there exists a maximum allowable jerk limit. This directly limits the rate at which the elevator can be brought to rest or to a constant speed and in turn restricts the minimum-possible shuttle time. For these reasons, a desired trajectory profile of position, which takes the jerk limit into consideration, is constructed for going from one location to the other as shown in Fig. 3(b) (for details on the trajectory design, refer to Beldiman et al., 1998). In the elevator industry, standardized tests for vertical motion are based on a one-shuttle run, i.e. when the elevator car is directed to transit from a given floor position to the next level. For ultra-high rise, on account of changing dynamics, it is necessary to simulate the performance at different car positions because the dynamics at the bottom is significantly different from a middle or top floor. One of the reasons why a test for one-shuttle run is preferred is because the constraints on the acceleration and jerk bear the most significance in this mode of operation. The elevator control system that regulates the car position is required to closely follow the reference trajectory. The shuttle time is defined by the amount of time it takes the car to settle to within 6 mm of the floor level. As seen from the trajectory profile, the objective is to attain a shuttle time of less than 6 s. It is also required that the elevator vertical ride be smooth enough not to violate the jerk and acceleration limits.

Both of these performance criteria are such that only the local dynamics around a certain floor matters, which can be approximated by a linear system. Re-leveling requires only guaranteed local performance at every location while for a one-shuttle run, the dynamics changes by an insignificant amount. The changing dynamics arising from non-linearities are factored into control design in such a way as to deliver required performances at *different* floors and to guarantee the non-linear stability (about which, more will be said in Section 3.2). Therefore, in order to meet these performance requirements, it is acceptable to consider only LTI control systems. The objective of the control system will be to meet these performance requirements in the face of parametric and non-parametric uncertainty. To evaluate the performance in the face of uncertainty, re-leveling and one-shuttle runs are simulated for different values of the parameter, θ, which are the extreme points. Although, this is often good enough, it is not exhaustive. To guarantee robust performance, the problem is formulated as a μ robust performance analysis (see Zhou, Doyle, & Glover, 1996) problem and robust performance is formally guaranteed. This is briefly illustrated by appealing to Fig. 4. In the bottom of Fig. 4, G is a nominal system (nominal parameters, θ_0 in Eq. (1)) that has real inputs, torque, position reference and payload, and fictitious inputs that account for the uncertainty arising from parametric and non-parametric errors. There are fictitious outputs corresponding to the fictitious inputs. The other outputs correspond to performance measures, such as the deviation of position from the dictated trajectory and re-leveling error, and sensor measurements such as drive sheave and car velocity. Once the control loop is closed, the nominal closed loop system, M, has only exogenous inputs and

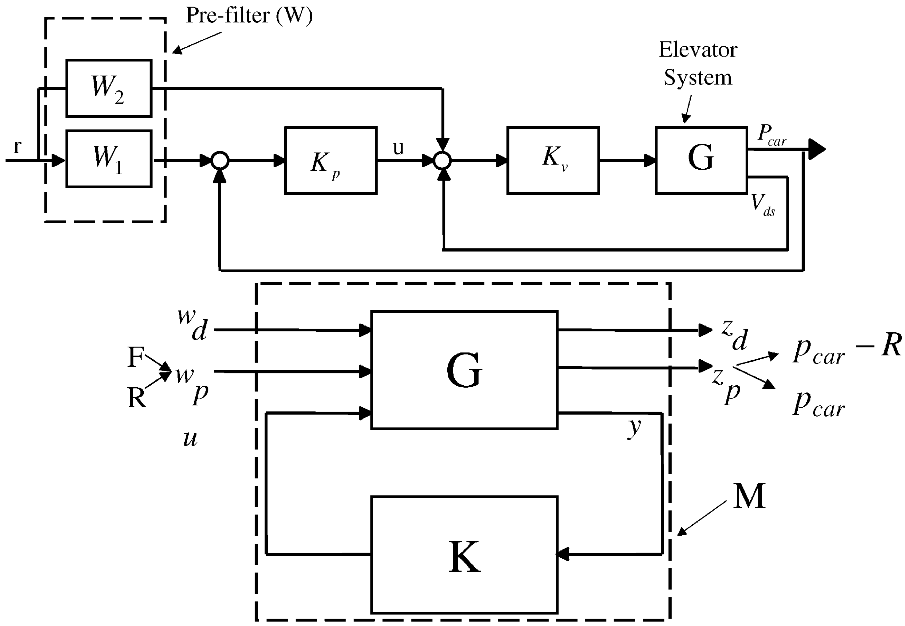

Fig. 4. Top: block diagram a sequential control setup; bottom: schematic diagram of the framework for μ robustness analysis.

outputs. The performance robustness of a nominal closed-loop system can be evaluated, based on a measure that is only a function of M, and thus, in turn, characterizes the performance of the candidate controller, K. The measure $\mu(M)$ is defined as

$$\mu(M) = \frac{1}{\max\{\|\Delta(j\omega)\|_2 \leqslant 1 | \det(I - \Delta M(j\omega)) \neq 0\}}$$

and performance is guaranteed if the value of μ is less than one (the reader is referred to Balas, Doyle, Glover, Packard, & Smith, 1995 and Zhou et al., 1996, for more details).

3.1. Tunable controller

The objective in this section is to develop a controller that not only meets robust performance requirements but is also readily tunable so that it can be customized for different hoistway heights. There are a larger number of methods that address the problem of control synthesis for robust performance (see Zhou et al., 1996; Diaz-Bobillo & Dahleh, 1992). However, all of these procedures, in general, lead to order inflation. Thus, it is not uncommon to apply such a control-synthesis tool and obtain a control system of extremely high order. For instance, for the nominal system of 54th order, the order of the controller could reach as large as 100. The problem with such high-order controllers is that it is difficult to gain much insight from the control law. The control law does not lend itself to tuning or customization for different hoistways. Faced with this situation, a design based on engineering intuition is developed. The design will be presented in the sequel, which relies on several observations and simplifications.

3.1.1. Redundancy

Re-leveling is an issue only when the elevator is stationed at any floor. To a first degree of approximation, only the rope-segment between the drive sheave and elevator cab stretches. The effect of payload force is to impose an equivalent-position disturbance and this effect is captured by vertical ride-performance measure. In other words, the controller naturally overcomes the effect of payload in an effort to drive the car to follow the desired trajectory under the presence of disturbance. Therefore, it is not necessary to explicitly take into account, the effect of payload, when designing the controller.

3.1.2. Design for the bottom of the hoistway

As shown in Fig. 1, the low-frequency lightly damped modes become more stable and shift to higher frequencies with higher car locations. Therefore, the controller is designed for the bottom.

3.1.3. Sequential control

By disregarding the payload signal, the problem is reduced to a SIMO system with a reference command input and two sensor outputs. Exploiting the structure of the dynamics makes it possible to design the controller "one-loop at a time" as shown at the top of Fig. 4, where K_v is the drive-sheave velocity-control loop, K_p is the car position-control loop and W is a feedforward controller, employed to attain quick response. The reasoning follows from the fact that the rope modes are not visible when one looks at the transfer function from torque to drive sheave and therefore, one cannot robustify against parametric and non-parametric uncertainties in the rope modes. Motor uncertainties, on the other hand, are better handled by the drive-sheave loop. The rope modes are, in turn, handled by the position-control loop.

3.1.4. Drive-sheave control

A PI loop is traditionally designed for the drive-sheave control loop. The point is that the rope modes do not play a significant part in the dynamics between the motor torque and the drive-sheave velocity. Moreover, since this problem has been addressed from several contexts—robustness, performance and bandwidth—both in the literature (see Suzuki et al., 1996; Watanabe et al., 1992) and in practice, there is no reason to modify the control system.

Now, it is necessary to design the pre-filter, W, and the car position-control system. Let the transfer function between u and P_{car} (when the drive sheave loop or the $K_v$loop closed) in Fig. 4 be given by P. Fig. 5 shows the Bode plot of the transfer function P generated with the identified model. The transfer function, P, drops phase by about 180° at a frequency, f_0, which amounts to the first rope mode. Since the uncertainty is parametric, the range of parametric variations, $\theta \in \Theta$, in amplitude and phase variations at the first rope mode is confined to a narrow band around the frequency, f_0. However, the phase always drops by around 180°, as also seen in Fig. 5. Now the closed-loop bandwidth required to meet the performance exceeds this frequency. This is coupled with the fact that it is required to maintain smooth performance across the range of parametric variations. It is well known (see Doyle, Francis, & Tannenbaum, 1991) that the loop gain needs to be large in the neighborhood of the variation, to be insensitive to the changes in the transfer function. However, it is impossible to simply have a large proportional gain, since there is a phase crossover and this will lead to instability.

The proposed idea is to employ a non-minimum phase all-pass filter for K_p:

$$K_p = \frac{a_1 s^2 - b_1 s + 1}{a_2 s^2 + b_2 s + 1}.$$

The notch is chosen at a frequency, f, that is close enough but smaller than the frequency, f_0, but such that there is significantly less variation at the frequency, f. The idea is to drop the phase from $-90°$ to (say) $-270°$ in the neighborhood of frequency, f, accompanied by a 15 db reduction in the loop gain at f, so that the closed-loop encirclement of -1 does not occur. Now, the poles of such a transfer function can be suitably placed at a frequency, which is larger than f_0. In this way, the loop gain in the neighborhood of frequency, f_0, can be increased by around 15 db without suffering the consequences of stability. This is because a 180° drop, on account of the rope mode, from a point where the phase is roughly 270° (which occurs on account of the notch) does not result in encirling the -1 point in the Nyquist plot. In this way, robustification can be achieved. The purpose of the pre-filter, W, as shown in Fig. 4, is to compensate for the notch, at frequency f. However, this is easy because the closed-loop transfer function does not change significantly here. The reader is encouraged to reflect upon why an alternate strategy such as a unity feedback for the car position followed by a pre-filter with lead lag at f_0 does not do a good job. This is because the parametric variations will now

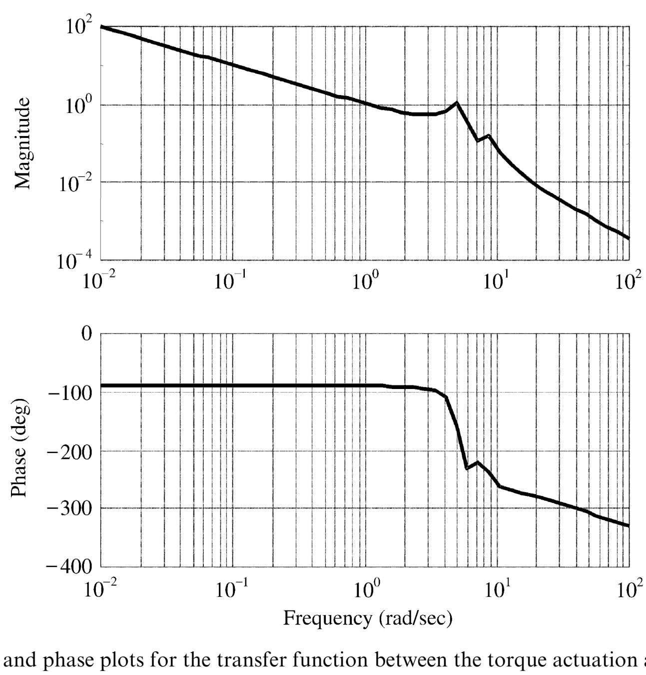

Fig. 5. Frequency domain magnitude and phase plots for the transfer function between the torque actuation and car position when the drive sheave loop is closed.

appear undiminished in the closed-loop elevator system. In summary, this strategy works because the structure of the uncertainty has been exploited. The variation in the transfer function occurs primarily due to real-parameter uncertainty. This necessarily implies that as the stiffness and damping varies, phase, as a function of frequency in this region, will have similar shape (since the number of modes will not change).

Now it is described how to customize the controller to different hoistways. A random input test is performed with the elevator car lodged at the bottom of the hoistway and the transfer function from the velocity of drive sheave to elevator car is obtained. This will help us determine the frequency of the first rope mode. Next, K_p is chosen as follows. The value of a_1 directly impacts the notch frequency and is chosen such that it is 5–10% less than the first rope mode. The ratio a_1/a_2 controls how well the parametric variations are damped and a value of 10 implies a de-sensitization of up to 90%. The ratio a_1/b_1 controls the notching effect and ranges from 3–5. These values can be used to determine K_p. The pre-filter, W, is a lead lag filter with lead term synchronized to the notch frequency. In this way, the controller can be customized for any hoistway.

The non-minimum phase controller is now evaluated on the standardized set of tests—re-leveling and vertical ride—that were presented earlier in this section. The simulations are based on the closed-loop performance with the lumped-parameter model of Eq. (1) serving as the model for the actual elevator system. As mentioned earlier, the performance is evaluated across the entire parameter range of $\theta \in \Theta$. For the re-leveling performance, the payload disturbance is applied to the system and the position trajectory is simulated, which is shown at the bottom left of Fig. 6. For evaluating vertical ride quality, the reference trajectory profile as seen at the bottom of Fig. 3(b) is adopted. The reference trajectory commands the elevator to transit by exactly one floor. The position error has been plotted in millimeters at the top left of Fig. 6. Remarkably, the performance shows a high degree of robustness to parametric variations. For both the plots in the figure, the x-axis is time (in s) and the y-axis is in mm. In the case of ride quality (tracking), only the trajectory for the last 10 mm from the target level is plotted. It is seen that it takes around 6 s for one shuttle run which is considered very good in the elevator industry. The trajectory is remarkably smooth to parametric variations.

However, as remarked earlier, these evaluations do not guarantee robust performance because the parameters chosen for simulation are restricted to the extreme points of the set Θ. A μ robust performance analysis is therefore performed, which is shown at the right of Fig. 6. Here x-axis denotes the frequency and the y-axis denotes the value of μ.

It is observed that the system satisfies robust performance because the maximum value of μ is smaller than one. Although the plots are shown only for the bottom floor, the results hold equally well for other floors as well.

3.2. Non-linear stability

So far, it has been shown that the elevator system is stable at each floor, which only proves local stability. In general, local stability at every operating point does not imply global stability and there are a number of examples in the literature that illustrate this point (see Slotine and Li (1996) for instance). Unlike, typical cases of non-linear stability problems, wherein the goal is to

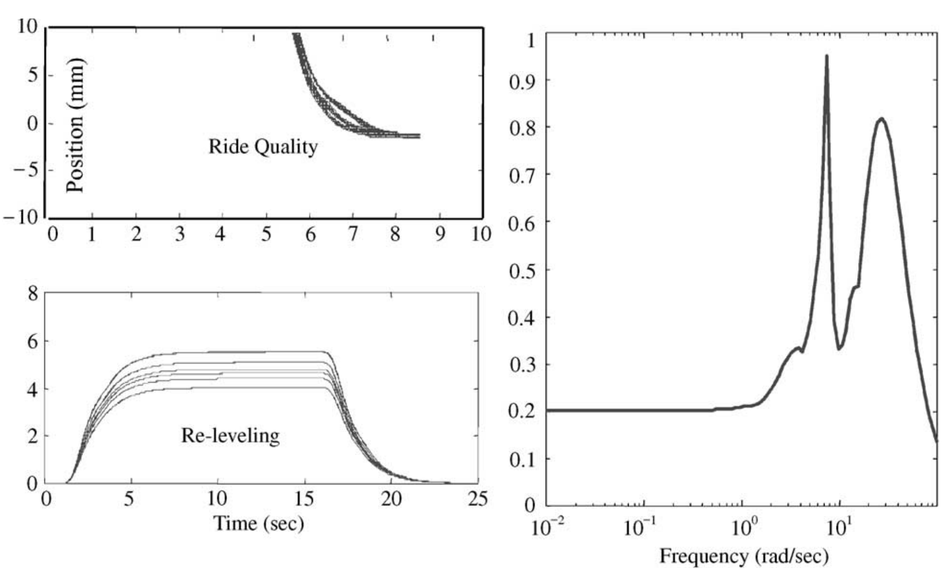

Fig. 6. Top-left: simulation of vertical ride of the elevator system as a function of time and with parametric variation; bottom-left: simulation of re-leveling response of the elevator system as a function of time and with parametric variation; right: μ robustness analysis plot.

prove stability around an equilibrium point, the difficulty here is that it is necessary to prove stability around every conceivable trajectory (for 100 floors, this amounts to about 10^4 trajectories). Therefore, the problem needs to be simplified and this will be presented in the sequel.

It is first shown that the trajectory-following problem can be reduced to a set-point regulation problem that can be further reduced to the question of absolute stability of an autonomous system. By approximating the non-linearity by means of a polynomial, the problem can be reduced to a Lure problem and the standard multi-variable Popov criterion can be applied to verify the non-linear stability numerically.

To this end, the nominal closed-loop system is written as follows:

$$\dot{x} = \begin{pmatrix} \dot{x}_p \\ \dot{x}_k \end{pmatrix} = A(x_p^1)x + Br \begin{bmatrix} A_p(x_p^1) & B_p(x_p^1)C_k^T \\ B_k\, C_p^T(x_p^1) & A_k \end{bmatrix} \begin{pmatrix} x_p \\ x_k \end{pmatrix} + B_k r, \tag{2}$$

where x_p and x_k are the states of the plant and the controller, respectively. The state, x_p, defined in Eq. (1), corresponds to the positions and velocities of all the inertial components. The first component, x_p^1, describes the car position. As pointed out in Eq. (1), the dynamics of the elevator system depend on car position. The matrices A_k, B_k, C_k characterize the state–space description of the LTI controller developed in Section 3.1.4. The signal, r, is the reference trajectory for taking the elevator from floor a to floor b. The objective is to prove that the states of the elevator system remain bounded on application of the signal r. Notice that if r were a small l_∞ bounded signal, robust control techniques can be readily used. This is because the problem can be formulated as a linear system with a small bounded non-linear perturbation. However, in this situation, these techniques are overly conservative and it is not possible to exploit the structure effectively.

With this in mind, consider the reference signal, r. In the elevator context, it is typically the case that the trajectory can be expressed as a filtered step input, i.e.,

$$r(t) = \alpha_{ab}\, W\, v(t), \quad v(t) = \begin{cases} 0, & t \leqslant 0, \\ 1, & t > 0, \end{cases} \tag{3}$$

where, W, is a stable filter with DC gain of unity and the set of all reference trajectories are parameterized by the one-dimensional parameter, α_{ab}. Without loss of generality, the stable filter, W, can be incorporated into the controller. Therefore, it is acceptable to pay attention to reference trajectories given by arbitrary step inputs. In this way, the problem reduces to that of set-point regulation.

For the sake of simplicity, a representative case is considered when the elevator is directed to move from a lower floor a to an upper floor b by means of a step input of size α. Now, if the dynamics were held constant, the LTI controller derived in Section 3.1.4 satisfies the performance requirements and, in particular, has been shown to give a steady-state error of zero. In this regard, the following technical lemma is obtained on the existence of a steady state, which follows readily from linear systems theory.

Lemma 1. *Consider the LTI system given by*

$$\dot{x}(t) = A(\alpha)\, x(t) + B(\alpha\, v(t)),$$

where A, W *and* B *are as in* Eq. (2), α *is the step input applied to take the elevator from the bottom to the* αth *floor, and* v *is a step input as in* Eq. (3). *It follows that* $x(\infty) = \lim_{t\to\infty} x(t)$ *and so*

$$A(\alpha)\, x(\infty) + B\alpha = 0.$$

Proof. The proof follows from linear systems theory and since it was showed in Section 3.1.4 that the closed-loop elevator system is stable, it tracks a step input with steady-state error equal to zero for all locations. Even though the lemma mentions only travel from floor 1 to the αthfloor, it should be clear that the lemma applies to any other traversal.

With this in mind, the coordinates are changed to focus on the error dynamics. Let $z(t) = x(t) - x(\infty)$. Then, the dynamics of z are described as

$$\dot{z} = A(z_1 + x_p^1(\infty))z + A(z_1 + x_p^1(\infty))\, x(\infty) + B\alpha,$$

$$z(0) = -\, x(\infty).$$

Applying Lemma 1, the dynamics can be re-written as

$$\dot{z} = A(z_1 + x_p^1(\infty))z + (A(z_1 + x_p^1(\infty)) - A(x_p^1(\infty)))\, x(\infty),$$

$$z(0) = -\, x(\infty).$$

Since $A(\cdot)$ is a well-behaved and matrix-valued function on a compact set, it can be arbitrarily closely approximated with polynomial functions. The compactness follows because the dynamics change only over a compact interval. When the elevator car is at the bottom, the rope length is at its maximum and there can be no further change in the rope modes. On the other hand, when the car is at the top of the hoistway, the rope ceases to play a part and the stiffness and damping are primarily due to the hitch and equalizer springs. Therefore, the above equation is re-written as

$$\Delta(z) = A(z_1 + x_p^1(\infty)) - A(x_p^1(\infty)) = \sum_{k=0}^{L} \Psi_k z_1^k, \quad \Psi_k \in R^{n\times n}.$$

Now in order to use the Popov criterion, it is necessary to re-formulate the problem as an interconnection of linear system with a memoryless decentralized non-linearity (see Khalil, 1992). It is claimed that the above equation can be re-written as follows, which will be shown in the sequel

$$\dot z = \tilde A z + \tilde B\,\Phi(y); \quad z(0) = -x(\infty),$$
$$y = (y_1, y_2, \ldots, y_m)^{\mathrm T} = \tilde C z, \tag{4}$$

where $\tilde A = A(x_p^1(\infty))$ and Φ is a decentralized non-linearity, i.e.,

$$\Phi(y) = [\phi_1(y_1), \phi_2(y_2), \ldots, \phi_m(y_m)]^T;$$
$$0 \leqslant \phi_i(y_i) \leqslant k_i y_i^2,\ k_i \geqslant 0, \quad i \in \{1, 2, \ldots, 2m\}. \tag{5}$$

From Fig. 2a, the first-order transfer function is evidently a good fit, i.e. $\Delta(z) = \Psi_{z_1}$, $\Psi \in R^{n\times n}$. Only this situation is considered here for the sake of simplicity. The general case follows similarly. Now it is shown that Eq. (4) holds. Consider the kth state equation with $(\tilde A)_k$, $(\tilde B)_k$ representing the kth columns. It follows that

$$\dot z_k = (\tilde A)_k z + \sum_{l=1}^{m} q_p z_1 z_l = (\tilde A)_k + \sum_{l=1}^{m} q_p (z_1 + z_p)^2 - (z_1 - z_p)^2;$$
$$q_l \in R, \quad l = 1, 2, \ldots, m$$

It should now be clear that the candidates for Φ, $\tilde B$, $\tilde C$ are

$$\phi_j(y_j) = y_j^2; \quad (\tilde C)_1 z = z_1, \quad \tilde C_{2j+1} z = z_1 + z_j,$$
$$\tilde C_{2j} = z_1 - z_j, \quad (\tilde B)_k = (q_1,\ q_2, -q_2,\ \ldots, q_m, -q_m)$$

The expression for non-linearity is only approximately correct. Furthermore, as written above, the non-linearity appears to be unbounded. What is actually true is that

$$\phi_j(y_j) = sat_{\alpha_j}(y_j^2),$$

where sat_α is a saturation function that saturates at amplitude, α. This expression is true because, first, there are no non-linearities in the controller and, second, the velocities always have limit switches that do not allow the elevator system to exceed a certain value. However, this fits in well with the Popov criterion because the non-linearities can be sector bounded and are still decentralized. In this way, the problem can be reduced to the Popov criterion which is described by the following theorem (see Khalil, 1992).

Theorem 1. *Consider the system in* Eq. (4) *and let A be Hurwitz, (A, B) controllable and (A, C) observable, $\Phi(\cdot)$ be a time-invariant decentralized non-linearity as in* Eq. (5). *Then the system is absolutely stable if there is an $\eta > 0$ with $-1/\eta$ not an eigenvalue of A, such that,*

$$Z(s) = I + (1 + \eta s)\, KG(s),$$
$$K = diag\{k_1, k_2, \ldots, k_{2m}\}$$

is strictly positive real.

On application of the theorem, it turns out that the k_i's describing the sector for each non-linearity, ranges anywhere between 0.1 and 200. This can be seen from Fig. 2 that the dynamics flatten out beyond the 60th floor, amounting to around 200 m (thus, the slope of the non-linearity can be bounded in a sector). It can be numerically shown that the Popov criterion does hold with $\eta \leqslant 2$, but unfortunately, the results cannot be plotted.

To get a feel for the result, a special case is considered when the first rope mode is the only one that is changing (this is the 1 Hz mode in Fig. 1). This mode gains significance only for the state corresponding to the car acceleration, $z_{car} = \dot z_1$. For this situation, the non-linear behavior as a function of car position has been plotted in terms of stiffness and damping in Fig. 2. The non-linearity can be described as

$$k_\infty \gamma_1 z_1^2 + c_\infty \gamma_2 z_1 z_{car} = k_\infty \gamma_1 z_1^2 + c_\infty \gamma_2 \frac{\mathrm d}{\mathrm dt} z_1^2,$$
$$-x_p^1 \leqslant z_1 \leqslant H_0 - x_p^1,$$

where k_∞, and c_∞ characterize the stiffness and damping at the target level. The constants γ_1 and γ_2 describe the constant rate of change of the stiffness and damping with car position z_{car}. Fig. 2 shows that γ_2 changes about 10% while γ_1 changes 5% over a range of 60 floors (equivalent to 200 m). Notice from Fig. 2 that the stiffness and damping start to flatten out beyond this height. This should not come as a surprise as the rope begins to have less effect with the equalizer spring accounting for a significant aspect of the dynamics. Now the above equation can be rewritten as

$$k_\infty \gamma_1 (1 + \gamma_\infty s)(z_1^2),\ \gamma_3 = \gamma_2 \frac{c_\infty}{k_\infty},$$
$$-x_p^1 \leqslant z_1 \leqslant H_0 - x_p^1,$$

where s is the Laplace transform and H_0 is the height up to which the first-order approximation holds. The problem can be cast as verifying the stability of a linear system, $G(s)(1 + \gamma_3 s)$ and a memoryless non-linearity given by $\phi(y) = y^2$, $y \in [-x_{pi}, H_0 - x_p^1]$, which saturates beyond this range. Notice that it is not really necessary to explicitly use the fact that $\phi(y) = y^2$, as long as the non-linearity belongs to the sector, which is all that matters. It turns out that the non-linearity belongs to the sector [0, 100]. The setup can be described by

$$\dot{z}(t) = \tilde{A}\, z(t) + B\, \phi(y),$$

$$y = z_1, \quad (6)$$

where B is a column vector which has zeros in all its rows except the row corresponding to the state characterizing the car acceleration. The stability of such a system is evaluated using the Popov criterion. It is necessary to verify whether there is an $\eta > 0$, with $k = 100$ such that

$$\Re((1 + j\eta w)(1 + j\gamma_3 \omega)\, k\, G(j\omega)) > 0.$$

That this is so, is seen graphically in Fig. 7, where $\eta \approx 10$ and this verifies the stability for this simple situation. It further implies that the steady-state error will converge to zero. This is because if the Popov criterion holds, a corresponding Lyapunov function can be formally constructed with its derivative less than zero along all the system trajectories. The reader is referred to Khalil (1992) for construction of such a Lyapunov function. It should also be seen that the non-linearity does not play a significant role in impacting stability in this specific situation and for the given controller. In this way, non-linear stability has been verified. It is worth pointing out that the Popov criterion is tested for different behaviors of stiffness and damping. As a behavior is enforced such that the stiffness increases faster than the damping, there comes a point when the system becomes unstable. Therefore, the fact that the property of the damping increasing faster than the stiffness in elevator systems, plays a critical role in maintaining non-linear stability. In this way, it is safe to gain confidence that a controller stabilized at the bottom of the hoistway should maintain non-linear closed loop stability.

4. Conclusion

This paper presents the design of control systems for vertical motion in ultra-high rise buildings. In contrast to low, mid- and some high-rise buildings, the ultra-high rises pose a unique problem for controlling vertical motion. The dynamics are characterized by low-frequency, lightly damped modes, which change as the elevator transits from one floor to another. A practical methodology has been developed for tunability and customizability of the controller to any hoistway. Features that are significant for control design are first defined, with the idea that these features can be identified readily by means of simple experiments. An LTI controller is designed, primarily based on these features, and can be readily customized and tuned to any hoistway. The simulations show that the controller delivers excellent vertical ride quality. In addition, it is insensitive to parametric changes that typically occur as a result of normal wear and tear. Finally, the issue of non-linear stability is confronted, that arises in an ultra-high rise system. It is shown that the fundamental behavior of rope modes with car position plays a critical role in maintaining non-linear stability for an LTI controller, designed for the bottom of the hoistway. In this way, a practical and functional methodology has been delivered for designing controllers for elevator vertical motion.

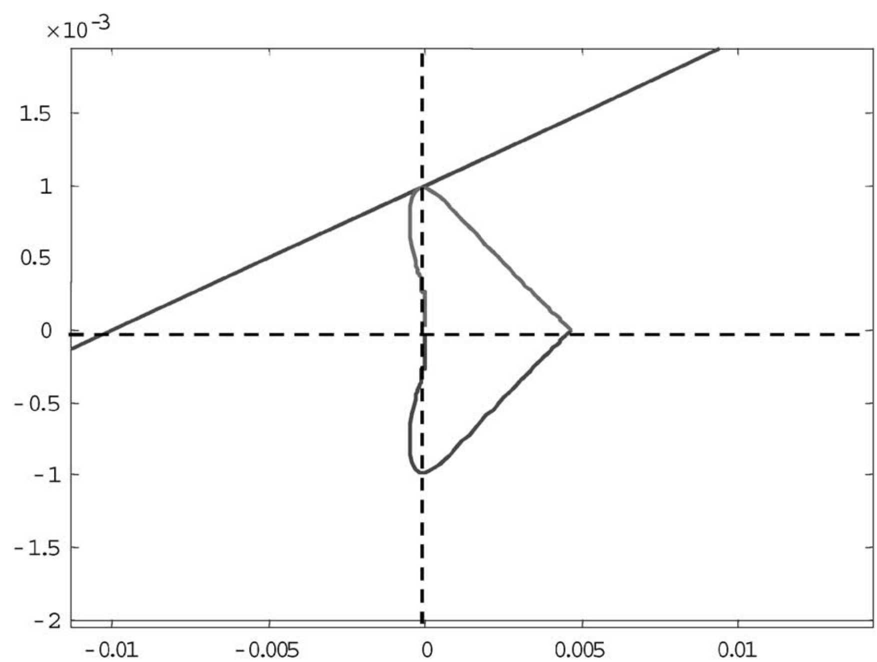

Fig. 7. Popov plot for the case where only the state equation for the car changes with position.

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